Optimal control of fed-batch processes with particle swarm optimization

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Abstract

Los problemas de control óptimo son normalmente descritos a través de sets de ecuaciones algebraicas y diferenciales no lineales. Son revisados algunos problemas de control óptimo en el área de bioprocesos. Utilizamos un optimizador de soluciones de nube de partículas para resolver una aproximación no lineal para el problema de control óptimo.

Keywords: Optimal control, particle swarm optimization

1. Introduction

A great number of valuable products are produced using fermentation processes and thus optimizing such processes is of great economic importance. Fermentation modeling process involves, in general, highly nonlinear and complex differential equations. Often optimizing these processes results in control optimization problems for which an analytical solution is not possible. By reformulating the optimal control problem as a non-linear, non-differentiable, optimization problem we are able to solve it with a derivative free optimization technique. We also propose a numerical environment to address the optimization of a fermentation process using available software for non-linear optimization and a development of an external library to handle the dynamic equations resulting from the complex differentiable equations.

Particle swarm [4, 2] was the technique used to solve the resulting non-linear approximation problems and proves to be competitive in finding a problem solution.

2. The optimization problem

The optimal control problem (P) can be posed as

$$\max \ J(t_f) \tag{4.69}$$
$$s.t. \quad \dot{x} = f(x,u,t) \tag{4.70}$$
$$\underline{x} \leq x(t) \leq \overline{x}, \forall t \in [t_0, t_f] \tag{4.71}$$
$$\underline{u} \leq u(t) \leq \overline{u}, \forall t \in [t_0, t_f] \tag{4.72}$$

where (4.70) is a set of differential equations, x are the state variables, u are the control variables and $J(t_f) = \varphi(x(t_f), t_f) + \int_{t_0}^{t_f} \phi(x, u, t)dt$.

φ is the performance index of the state variables at final time t_f and ϕ is the integrated performance index during the operation.

Imposing an infinity penalty for the (4.71) constraints violation results in replacing the objective function $J(t_f)$ by

$$\hat{J}(t_f) = \begin{cases} J(t_f) & \text{if } \underline{x} \leq x(t) \leq \overline{x}, \forall t \in [t_0, t_f] \\ +\infty & \text{otherwise.} \end{cases}$$

We address the functional constraints (4.72) by approximating functions $u(t)$ with linear splines. Let w_i, $i = 0, \ldots, n$ be the linear spline values at n equally spaced point in $[t_0, t_f]$. Constraint (4.72) can then be approximated by $\underline{u} \leq w_i \leq \overline{u}$.

The control optimization problems were coded in the AMPL [3] modeling language and an external dynamic library was made to compute the objective function values. The CVODE [1] library was used to solve the corresponding differentiable equations.

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4. Bibliography

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