Direct Multisearch for Multiobjective Optimization

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4 Further improvements on DMS

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Outline

1. Introduction and motivation
2. Direct MultiSearch
3. Numerical results
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Derivative-free multiobjective optimization

MOO problem

\[
\min_{x \in \Omega} F(x) \equiv (f_1(x), f_2(x), \ldots, f_m(x))^T
\]

where

\[
\Omega = \{ x \in \mathbb{R}^n : \ell \leq x \leq u \}
\]

\[
f_j : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}, j = 1, \ldots, m, \ell \in (\mathbb{R} \cup \{-\infty\})^n \text{ and } u \in (\mathbb{R} \cup \{+\infty\})^n
\]

- Several objectives, often conflicting.
- Functions with unknown derivatives.
- Expensive function evaluations, possibly subject to noise.
- Impractical to compute approximations to derivatives.
Derivative-free multiobjective optimization

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2. Direct MultiSearch
3. Numerical results
4. Further improvements on DMS
5. Conclusions and references
DMS algorithmic main lines

- Does **not aggregate** any of the objective **functions**.
- Generalizes ALL direct-search methods of directional type to MOO.
- Makes use of search/poll paradigm.
- Implements an optional search step (only to disseminate the search).
- Tries to capture the whole Pareto front from the polling procedure.
- Keeps a list of feasible nondominated points.
- Poll centers are chosen from the list.
- Successful iterations correspond to list changes.
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A.I.F. Vaz (CERFACS 2011)
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At each iteration considers a list of feasible nondominated points \( L_k \).

Evaluate a finite set of feasible points \( L_{add} \).

Remove dominated points from \( L_k \cup L_{add} \rightarrow L_{filtered} \).

Select list of feasible nondominated points \( L_{trial} \).

Compare \( L_{trial} \) to \( L_k \) (success if \( L_{trial} \neq L_k \), unsuccess otherwise).
DMS search & poll steps

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Numerical Example — Problem SP1 [Huband et al.]

- Evaluated points since beginning.
- Current iterate list.
Numerical example — problem SP1 [Huband et al.]

- Evaluated poll points.
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Refining subsequences and directions

For both globalization strategies (using the mesh or the forcing function in the search step), one also has:

**Theorem (existence of refining subsequences)**

There is at least a convergent subsequence of iterates \( \{x_k\}_{k \in K} \) corresponding to unsuccessful poll steps, such that \( \alpha_k \to 0 \) in \( K \).

**Definition**

Let \( x_* \) be the limit point of a convergent refining subsequence.

Refining directions for \( x_* \) are limit points of \( \{d_k/\|d_k\|\}_{k \in K} \) where \( d_k \in D_k \) and \( x_k + \alpha_k d_k \in \Omega \).
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**Definition**

Let \( x_\ast \) be the limit point of a convergent refining subsequence.

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Refining directions for \( x^* \) are limit points of \( \{d_k/\|d_k\|\}_{k \in K} \) where \( d_k \in D_k \) and \( x_k + \alpha_k d_k \in \Omega \).
Pareto-Clarke critical point

Let us focus (for simplicity) on the unconstrained case, $\Omega = \mathbb{R}^n$.

**Definition**

$x_*$ is a **Pareto-Clarke critical point** of $F$ (Lipschitz continuous near $x_*$) if

$$\forall d \in \mathbb{R}^n, \exists j = j(d), f^o_j(x_*; d) \geq 0.$$
Analysis of DMS

**Assumption**

- \( \{x_k\}_{k \in K} \) refining subsequence converging to \( x_* \).
- \( F \) Lipschitz continuous near \( x_* \).

**Theorem**

If \( v \) is a refining direction for \( x_* \) then

\[ \exists j = j(v) : f_j(x_*; v) \geq 0. \]
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Convergence analysis of DMS

Theorem

If the set of refining directions for $x^*$ is dense in $\mathbb{R}^n$, then $x^*$ is a Pareto-Clarke critical point.

Notes

- When $m = 1$, the presented results coincide with the ones reported for direct search.
- This convergence analysis is valid for multiobjective problems with general nonlinear constraints.
Convergence analysis of DMS

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If the set of refining directions for \( x_\star \) is dense in \( \mathbb{R}^n \), then \( x_\star \) is a Pareto-Clarke critical point.

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1. Introduction and motivation
2. Direct MultiSearch
3. Numerical results
4. Further improvements on DMS
5. Conclusions and references
Numerical testing framework

Problems

- Number of variables between 1 and 30.
- Number of objectives between 2 and 4.

Solvers

- DMS tested against 8 different MOO solvers (complete results available at http://www.mat.uc.pt/dms).
- Results reported only for
  - AMOSA – simulated annealing code.
  - BIMADS – based on mesh adaptive direct search algorithm.
  - NSGA-II (C version) – genetic algorithm code.

All solvers tested with default values.
Numerical testing framework

Problems

- 100 bound constrained MOO problems (AMPL models available at [http://www.mat.uc.pt/dms](http://www.mat.uc.pt/dms)).
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DMS numerical options

- No search step.

- List initialization: sample along the line $\ell - u$.

- List selection: all current feasible nondominated points.

- List ordering: new points added at the end of the list, poll center moved to the end of the list.

- Positive basis: $[I - I]$.

- Step size parameter: $\alpha_0 = 1$, halved at unsuccessful iterations.

- Stopping criteria: minimum step size of $10^{-3}$ or a maximum of 20000 function evaluations.
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Performance metrics — Purity

\( F_{p,s} \) (approximated Pareto front computed by solver \( s \) for problem \( p \)).

\( F_p \) (approximated Pareto front computed for problem \( p \), using results for all solvers).

Purity value for solver \( s \) on problem \( p \):

\[
\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}.
\]
Comparing DMS to other solvers (Purity)

Purity performance profile

Purity Metric (percentage of points generated in the reference Pareto front)

\[ t_{p,s} = \frac{|F_{p,s}|}{|F_{p,s} \cap F_p|} \]
Comparing DMS to other solvers (Purity)

Purity performance profile with the best of 10 runs

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Performance metrics — Spread

Gamma Metric (largest gap in the Pareto front)

\[ \Gamma_{p,s} = \max_{j \in \{1, \ldots, m\}} \left( \max_{i \in \{0, \ldots, N\}} \{ \delta_{i,j} \} \right) \]
Comparing DMS to other solvers (Spread)

Average $\Gamma$ performance profile for 10 runs

Gamma Metric (largest gap in the Pareto front)
Performance metrics — Spread

**Delta Metric** (uniformity of gaps in the Pareto front)

\[ \Delta_{p,s} = \max_{j \in \{1,\ldots,m\}} \left( \frac{\delta_{0,j} + \delta_{N,j} + \sum_{i=1}^{N-1} |\delta_{i,j} - \bar{\delta}_j|}{\delta_{0,j} + \delta_{N,j} + (N - 1)\bar{\delta}_j} \right) \]

where \( \bar{\delta}_j \), for \( j = 1, \ldots, m \), is the \( \delta_{i,j} \)'s average.
Comparing DMS to other solvers (Spread)

Average $\Delta$ performance profile for 10 runs

Delta Metric (uniformity of gaps in the Pareto front)
Comparing DMS to other solvers

Data profile with the best of 10 runs ($\varepsilon=0.05$)

- DMS(n, line)
- BIMADS
- NSGA-II (C version)
- AMOSA

# maximum function evaluations = 5000
Comparing DMS to other solvers

Data profile with the best of 10 runs (ε=0.5)

DMS(n,line)  BIMADS  NSGA-II (C version)  AMOSA

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Average Δ performance profile for 10 runs

Delta Metric (uniformity of gaps in the Pareto front)
Comparing DMS to other solvers

Data profile with the best of 10 runs ($\epsilon=0.05$)

$\sigma d_s(\sigma)$

- DMS(n,line)
- DMS(n,line,cache,spread)
- BIMADS
- NSGA-II (C version)
- AMOSA

$\#$ maximum function evaluations = 5000
Comparing DMS to other solvers

Data profile with the best of 10 runs ($\varepsilon=0.5$)

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Conclusions and references

- Development and analysis of a novel approach (Direct MultiSearch) for MOO, generalizing ALL direct-search methods.

- Direct MultiSearch (DMS) exhibits highly competitive numerical results for MOO.

DMS (Matlab implementation) and problems (coded in AMPL) freely available at: http://www.mat.uc.pt/dms.

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Conclusions and references

- Development and analysis of a novel approach (Direct MultiSearch) for MOO, generalizing ALL direct-search methods.

- Direct MultiSearch (DMS) exhibits highly competitive numerical results for MOO.

DMS (Matlab implementation) and problems (coded in AMPL) freely available at: http://www.mat.uc.pt/dms.