Extension of PSwarm to Linearly Constrained Derivative-Free Global Optimization

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SIAM Conference on Optimization

May 10-13, 2008

Outline

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Problem formulation

The problem we are addressing is:

Problem definition - bound constraints

$$\min_{z \in \mathbb{R}^n} \; f(z)$$
$$\text{s.t.} \quad \ell \leq z \leq u,$$

where $\ell \leq z \leq u$ are understood componentwise.

Smoothness – Assumption

To apply particle swarm or coordinate search, smoothness of the objective function $f(z)$ is not required.
For the convergence analysis of coordinate search, and therefore of the hybrid algorithm, some smoothness of the objective function $f(z)$ is imposed.

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Particle Swarm (new position and velocity)

The new particle position is updated by

Update particle

$$x^p(t+1) = x^p(t) + v^p(t+1),$$

where $v^p(t+1)$ is the new velocity given by

Update velocity

$$v^p(t+1) = \iota(t)v^p(t) + \mu\omega_1(t)\left(y^p(t) - x^p(t)\right) + \nu\omega_2(t)\left(\hat{y}(t) - x^p(t)\right),$$

where $\iota(t)$, μ and ν are parameters and $\omega_1(t)$ and $\omega_2(t)$ are random vectors drawn from the uniform $(0, 1)$ distribution.
$y^p(t)$ is the best particle p position and $\hat{y}(t)$ is the best population position.

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Some properties

- Easy to implement.
- Easy to deal with discrete variables.
- Easy to parallelize.
- For a correct choice of parameters the algorithm terminates ($\lim_{t \to +\infty} v(t) = 0$).
- Uses only objective function values.
- Convergence for a global optimum under strong assumptions (unpractical).
- High number of function evaluations.

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Introduction to direct search methods

- Direct search methods are an important class of optimization methods that try to minimize a function by comparing objective function values at a finite number of points.
- Direct search methods do not use derivative information of the objective function nor try to approximate it.
- Coordinate search is a simple direct search method.

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Some definitions

Maximal positive basis

Formed by the coordinate vectors and their negative counterparts:

$$D_{\oplus} = \{e_1, \ldots, e_n, -e_1, \ldots, -e_n\}.$$

$D_{\oplus}$ spans $\mathbb{R}^n$ with nonnegative coefficients.

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Given $D_\oplus$ and the current point $y(t)$, two sets of points are defined: a grid M_t and the poll set P_t.

Sets

The grid M_t is given by

$$M_t = \left\{ y(t) + \alpha(t) D_\oplus z,\ z \in \mathbb{N}_0^{|D_\oplus|} \right\},$$

where $\alpha(t) > 0$ is the grid size parameter.
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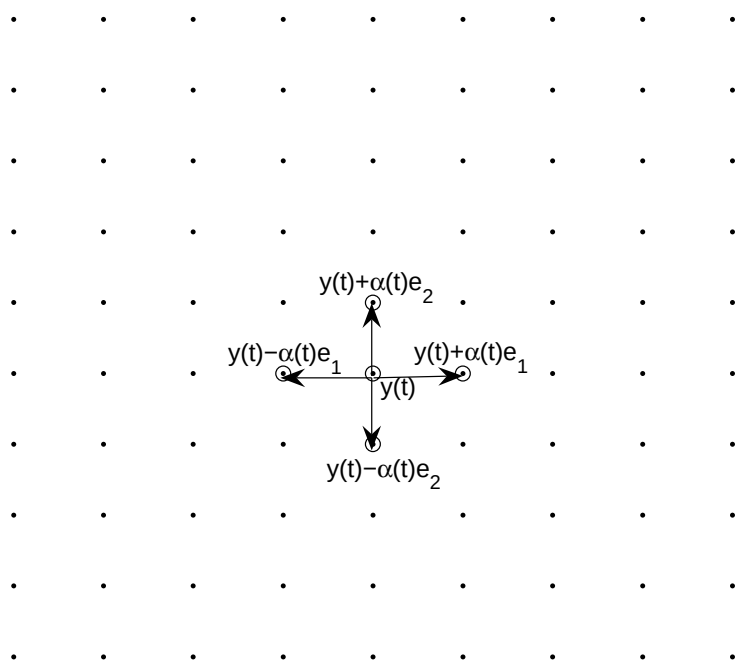

The grid M_t and the set P_t when $D_\oplus = \{e_1, e_2, -e_1, -e_2\}$.

Coordinate search

- The search step conducts a finite search on the grid M_t.
- If no success is obtained in the search step then a poll step follows.
- The poll step evaluates the objective function at the elements of P_t, searching for points which have a lower objective function value.
- If success is attained, the value of $\alpha(t)$ may be increased, otherwise it is reduced.

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Motivation for PSwarm

Central idea

A particle swarm iteration is performed in the search step and if no progress is attained a poll step is taken.

Key points

- In the first iterations the algorithm takes advantage of the particle swarm ability to find a global optimum (exploiting the search space), while in the last iterations the algorithm takes advantage of the pattern search robustness to find a stationary point.
- The number of particles in the swarm search can be decreased along the iterations (no need to have a large number of particles around a local optimum).

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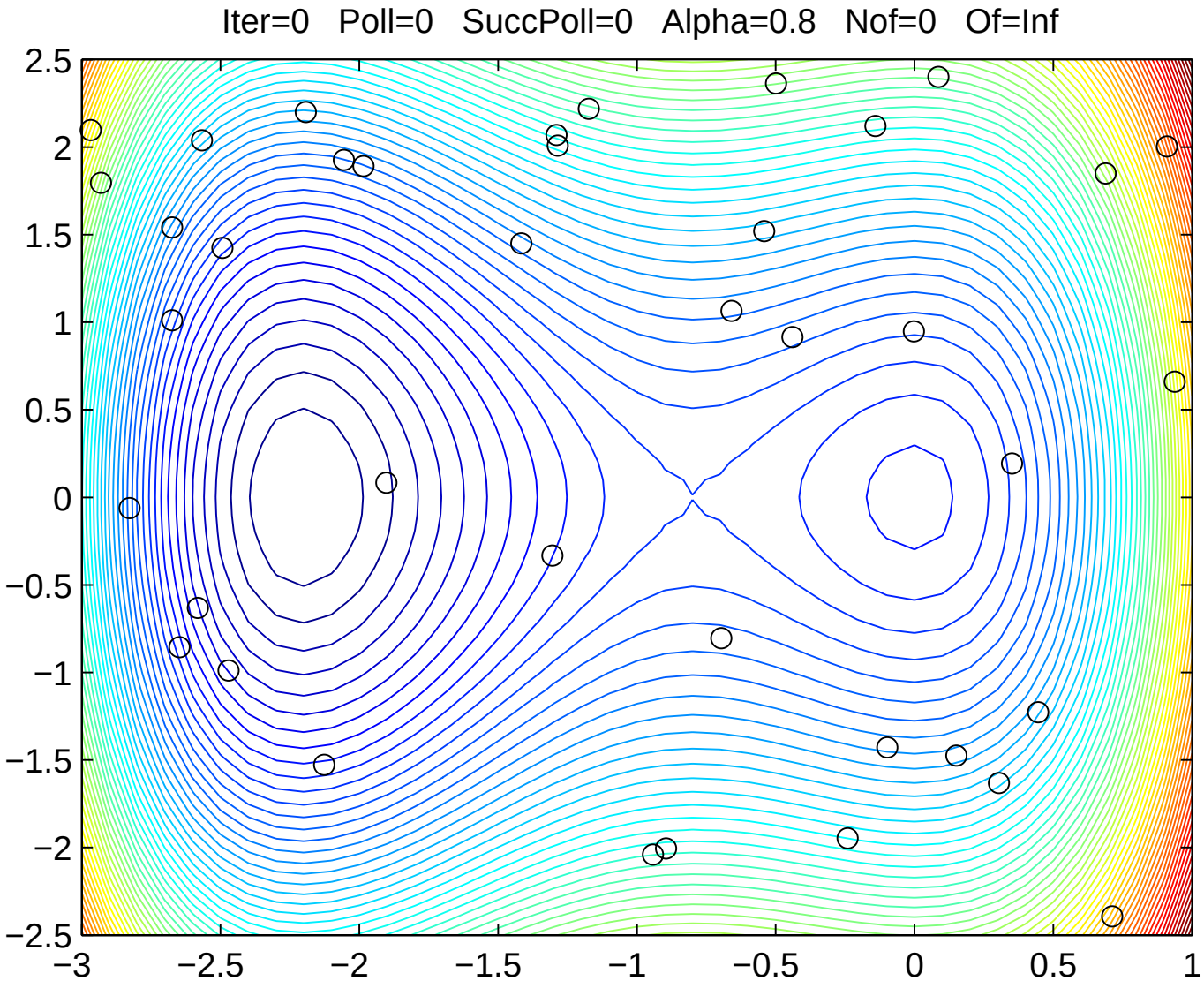
An example - Treccani function



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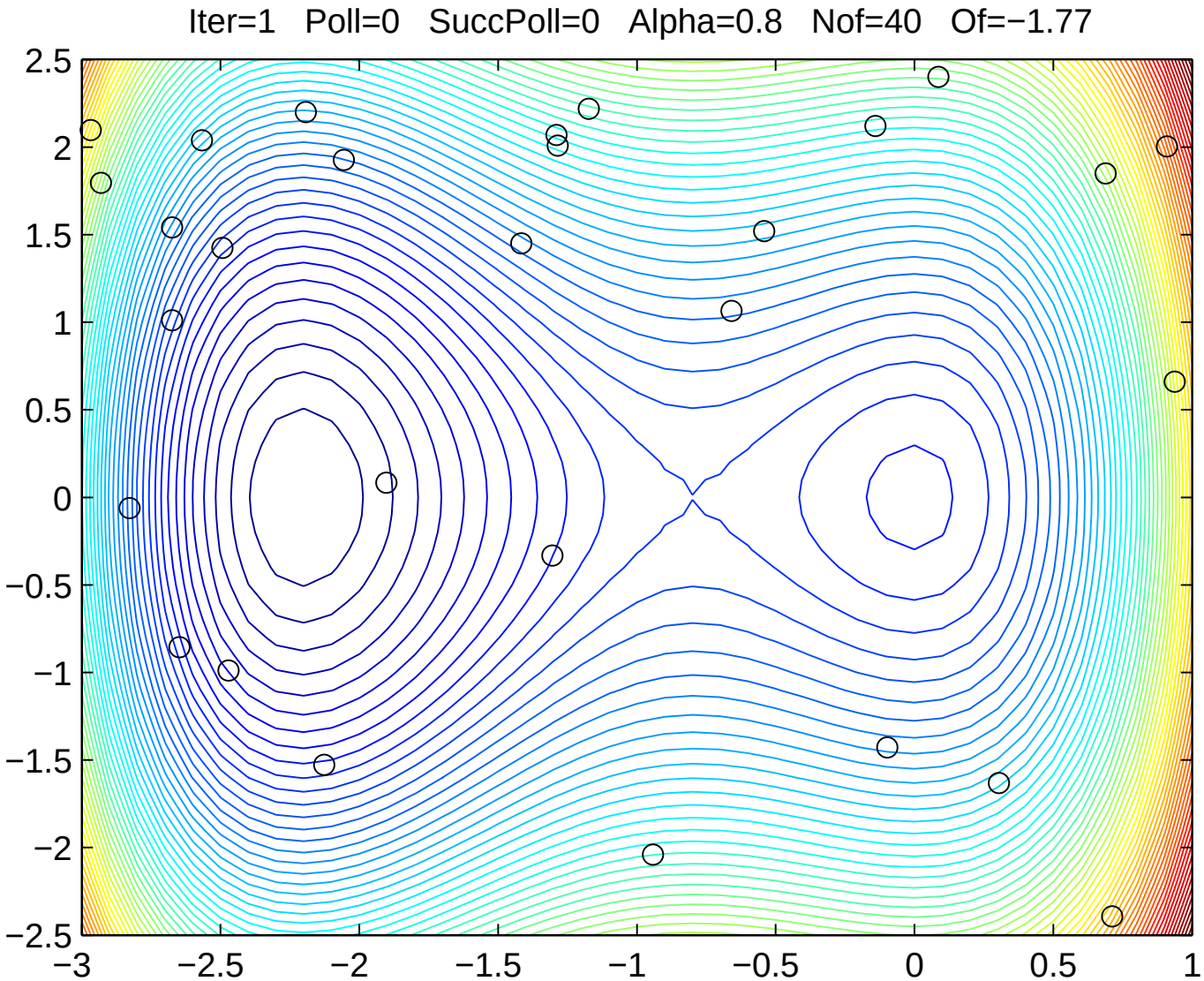

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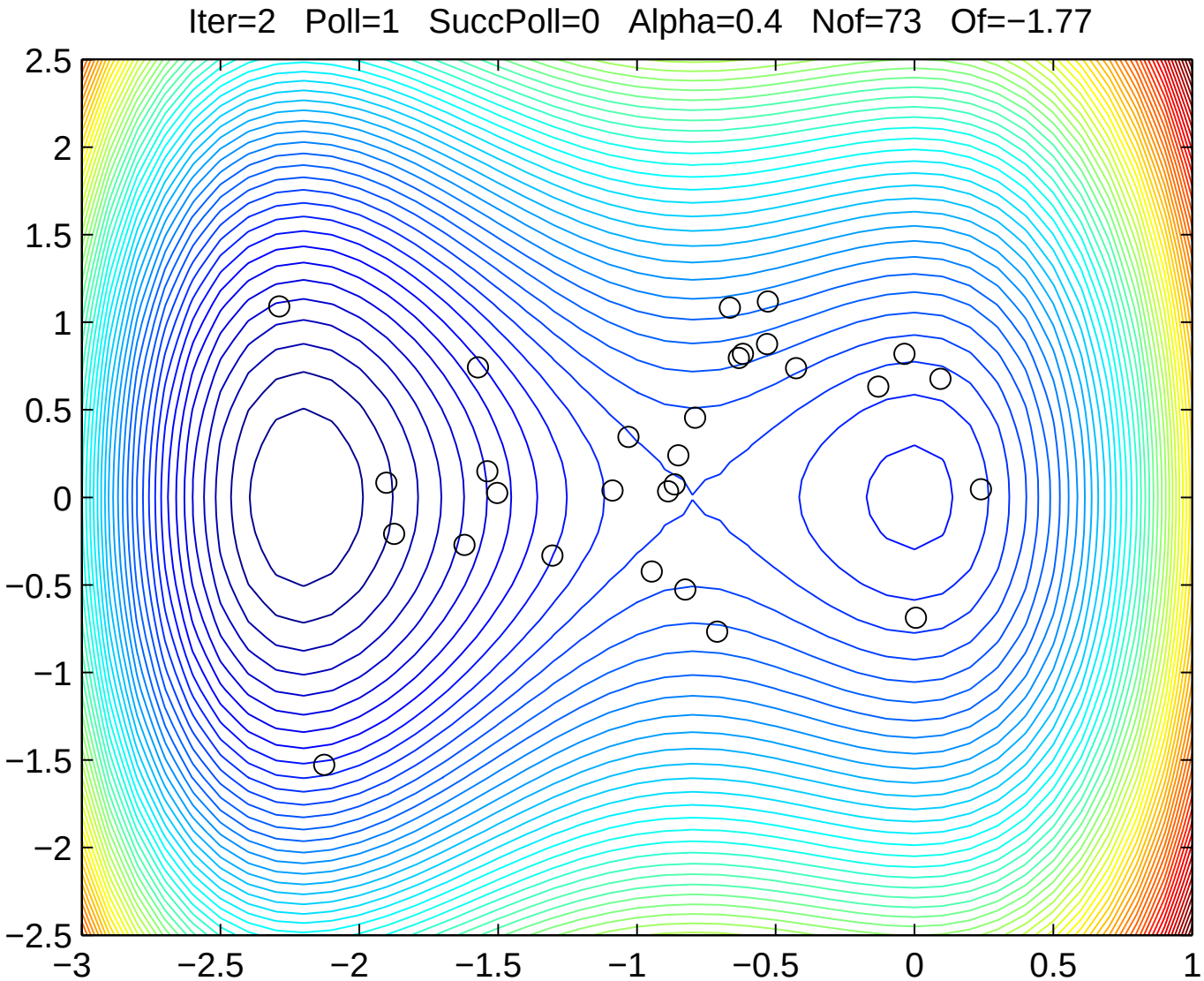

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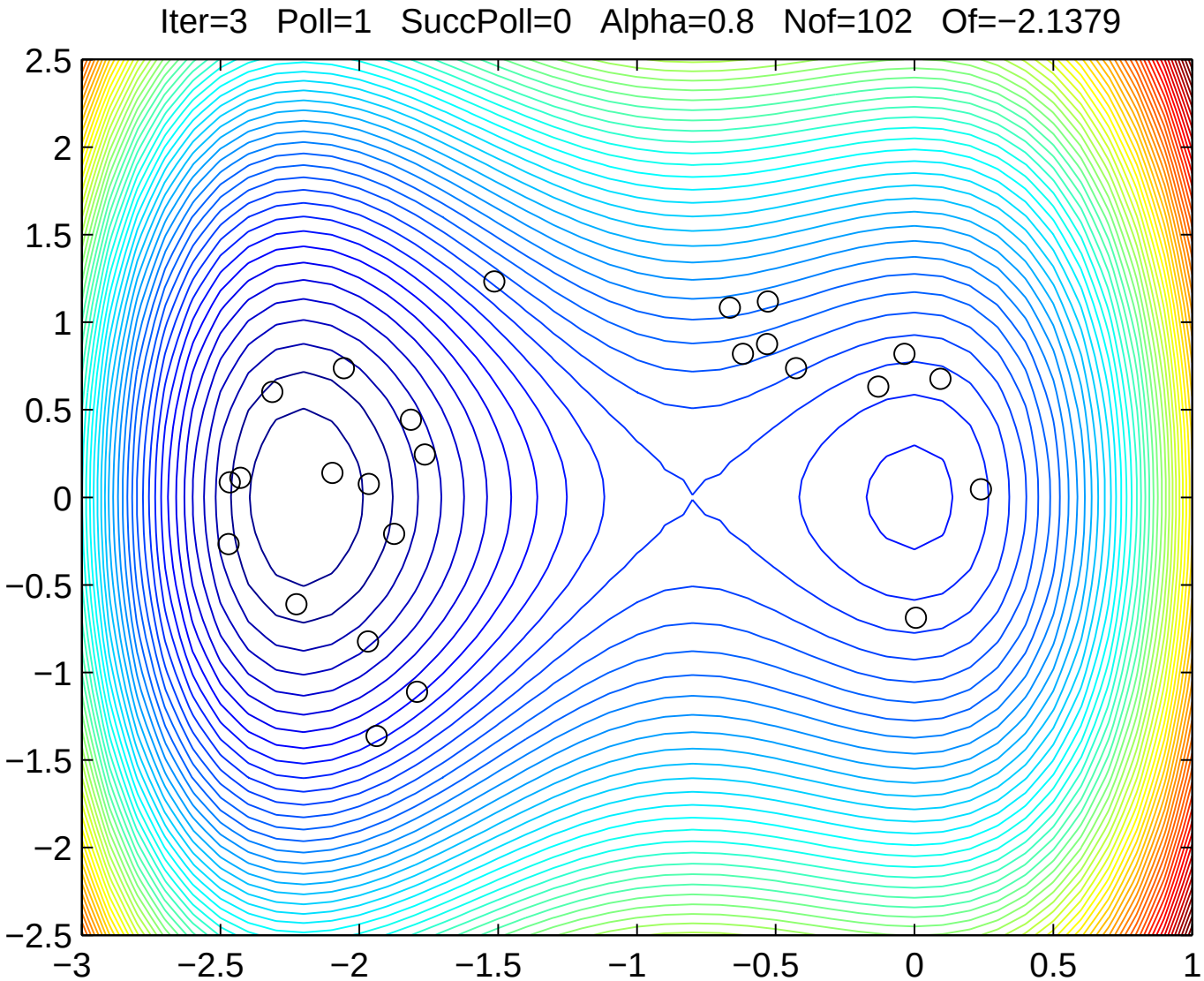

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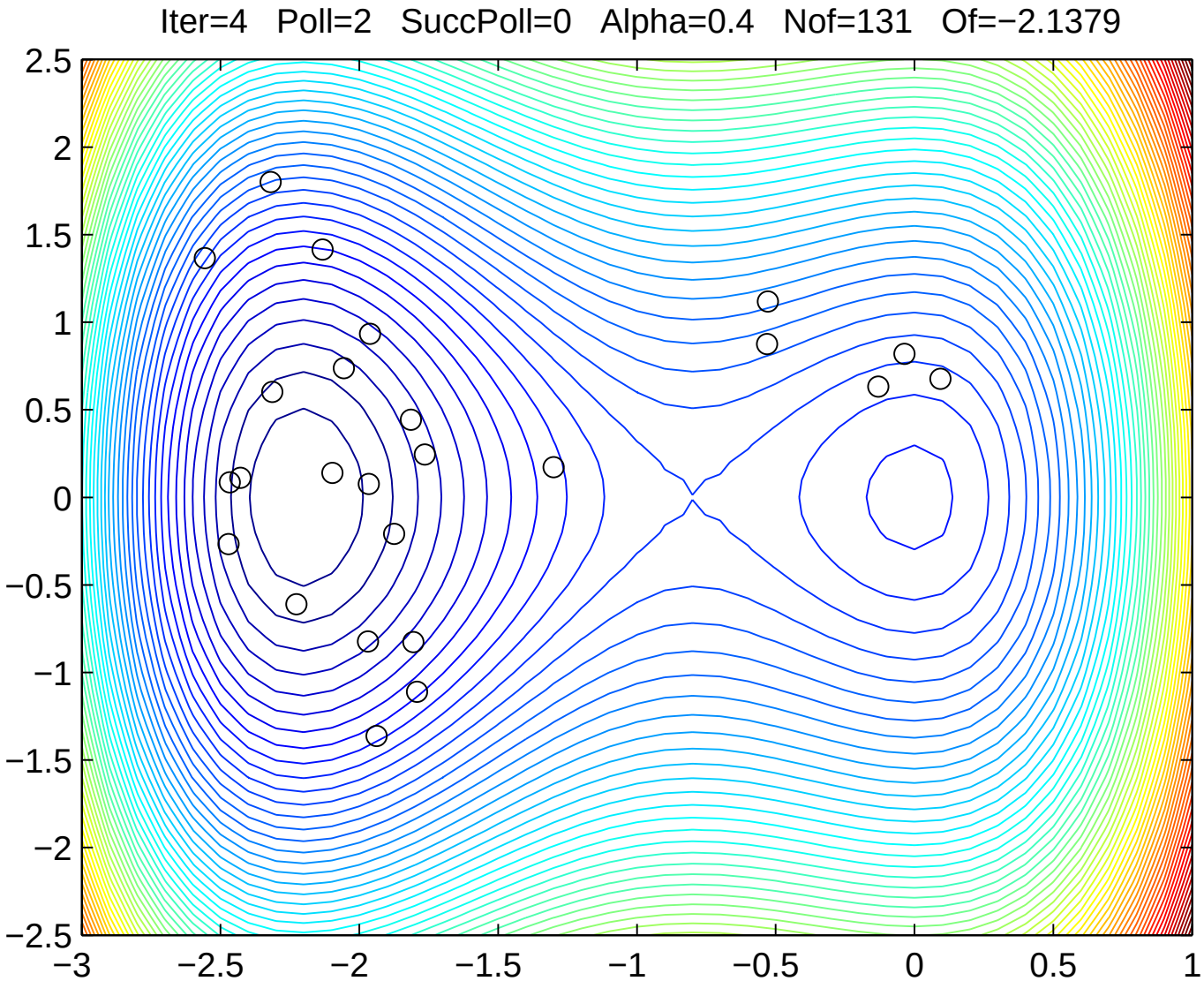

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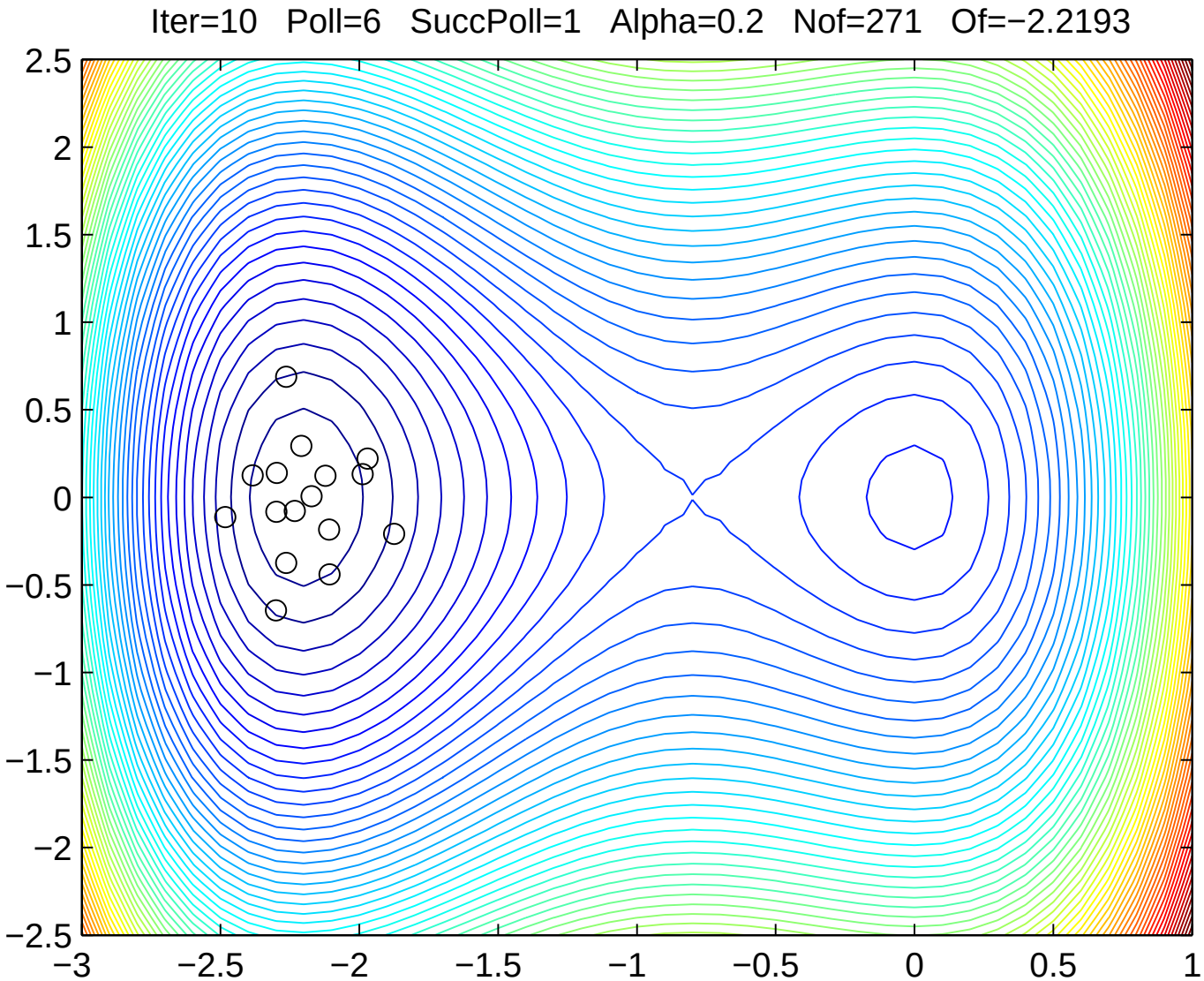

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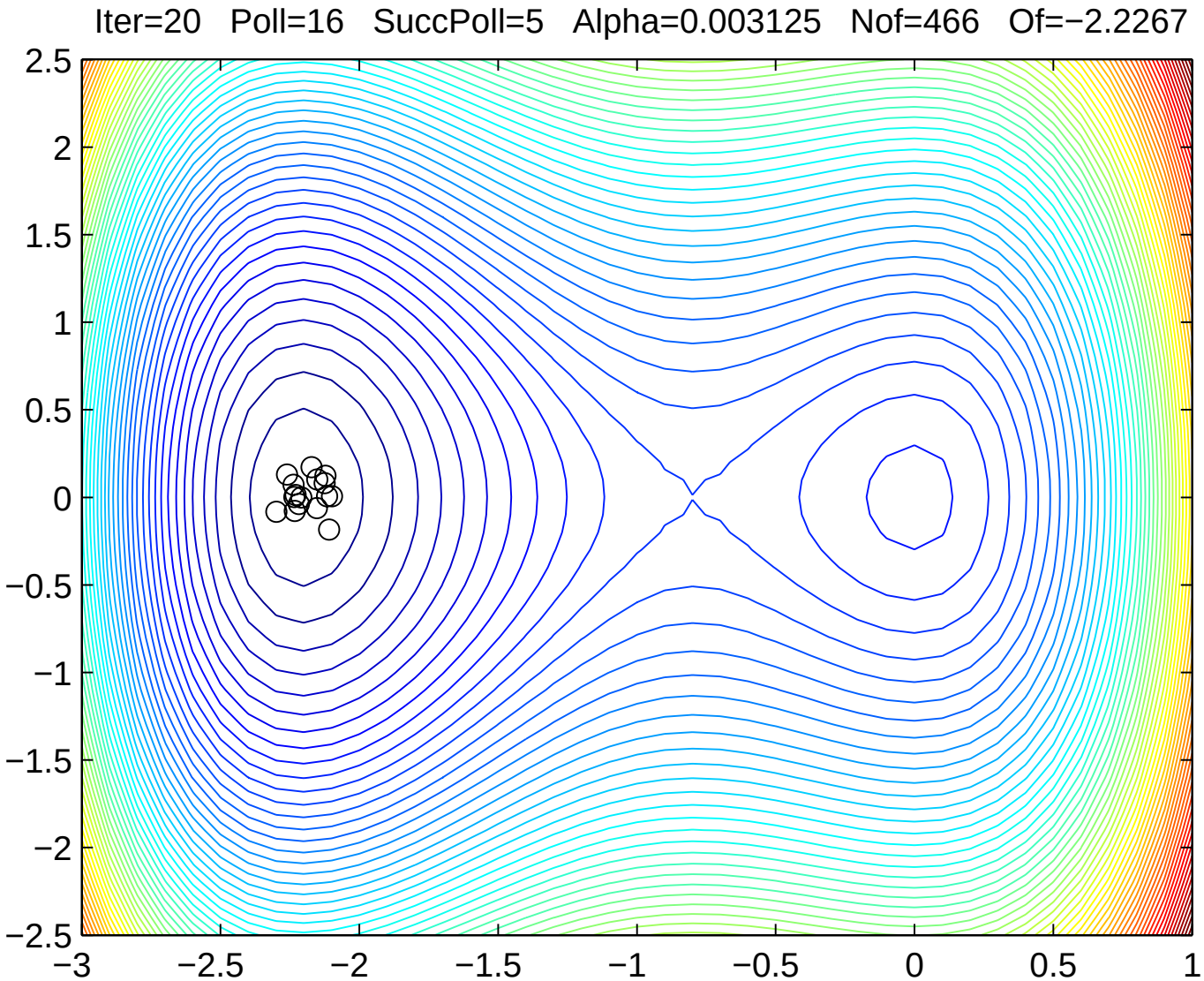

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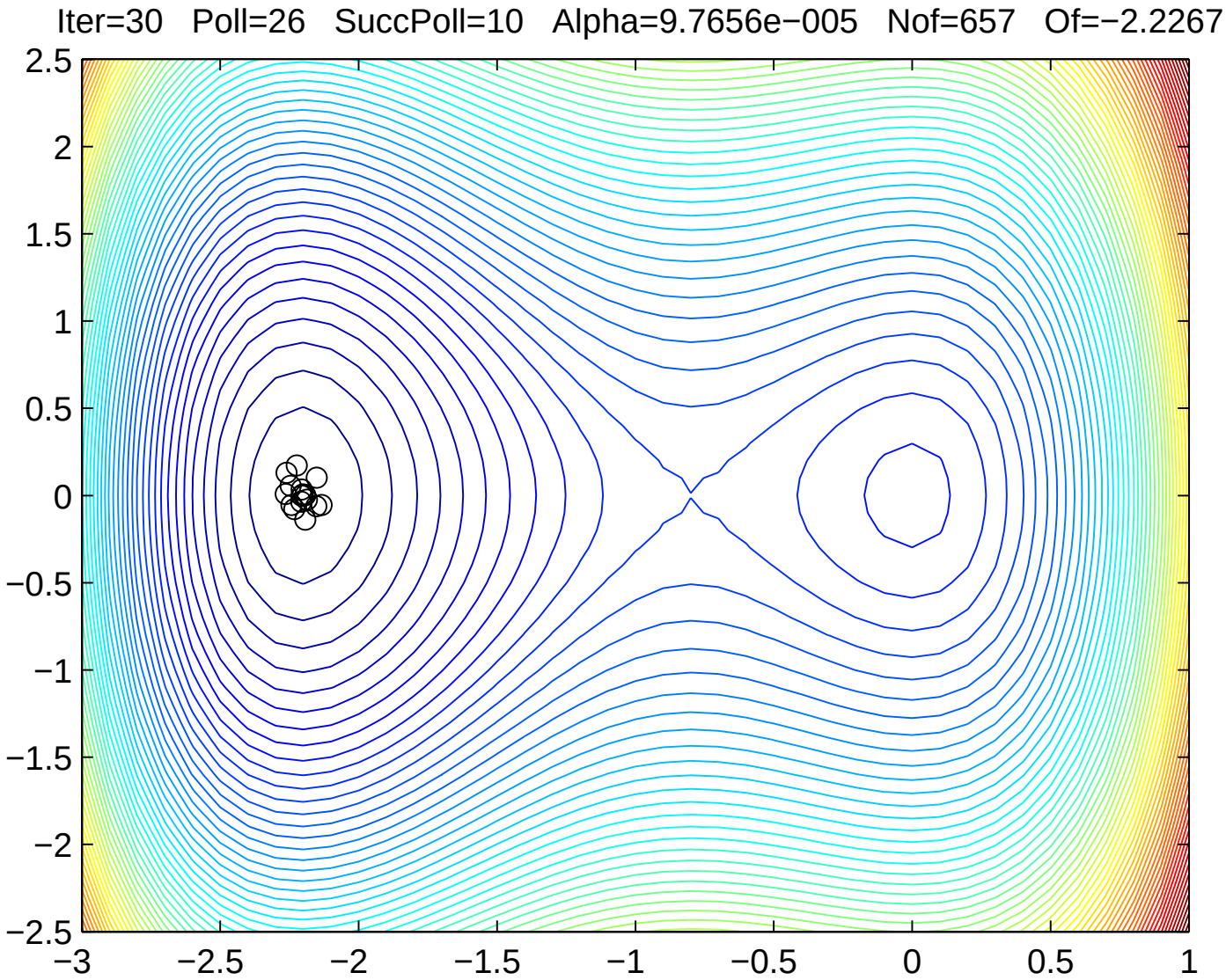

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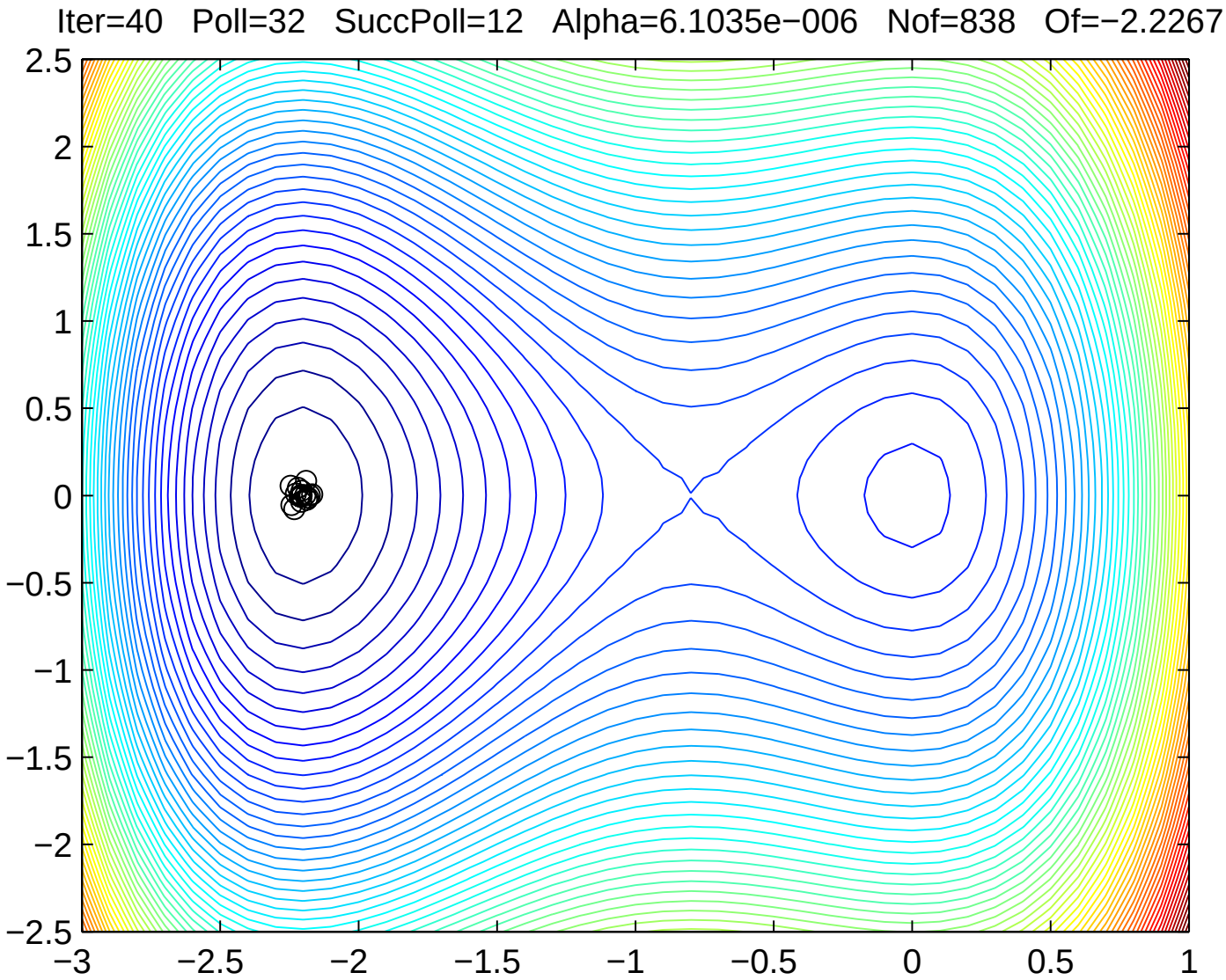

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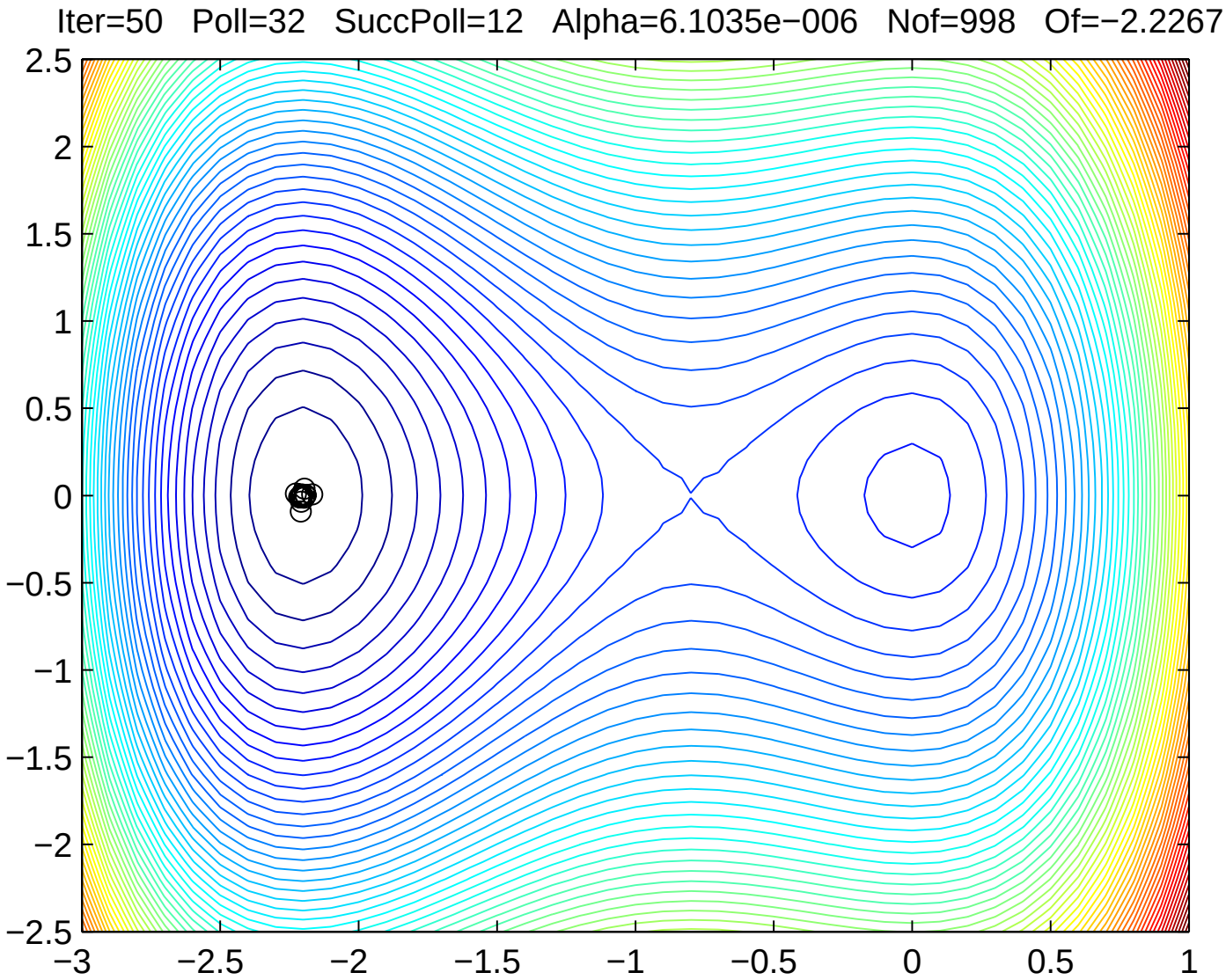

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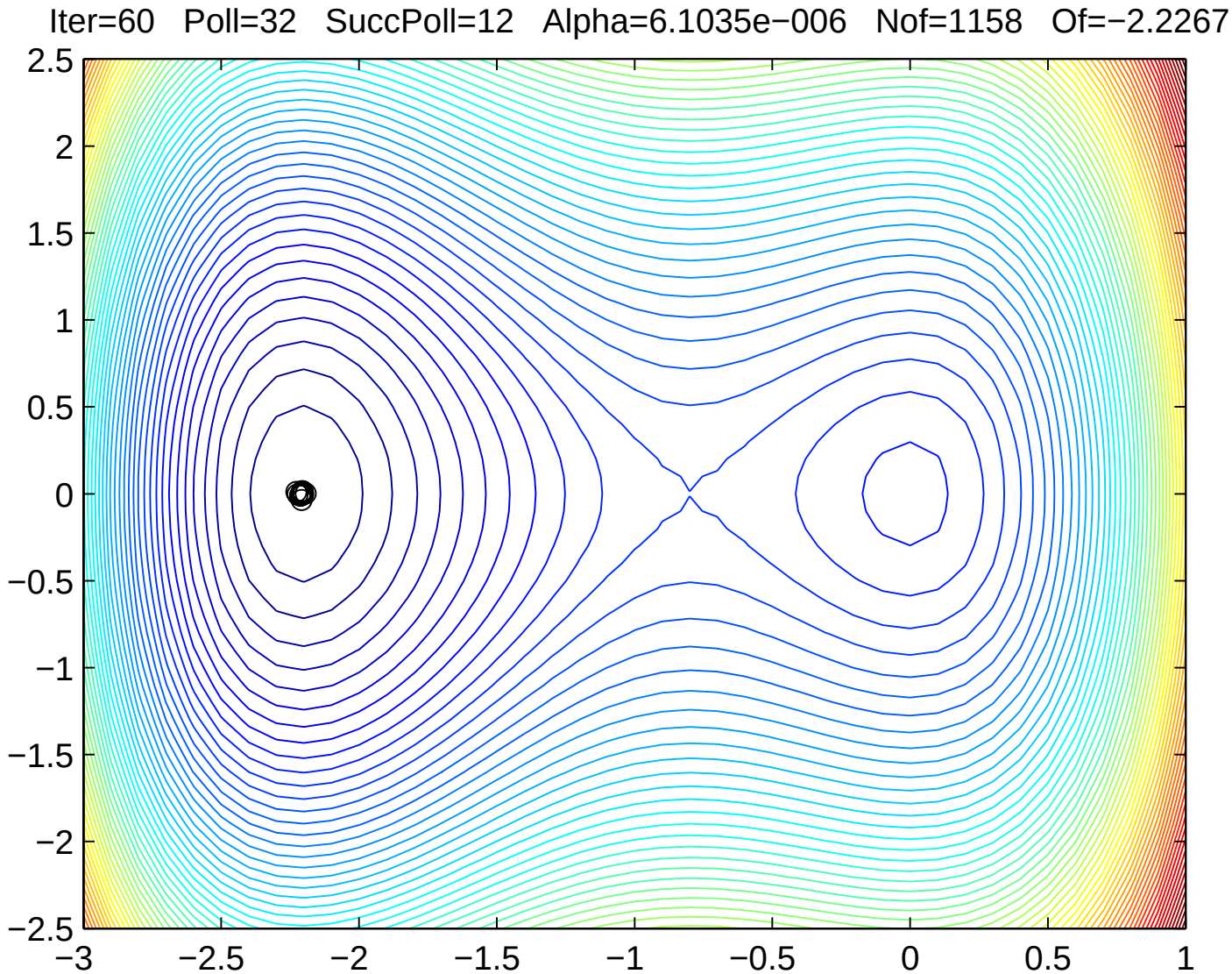

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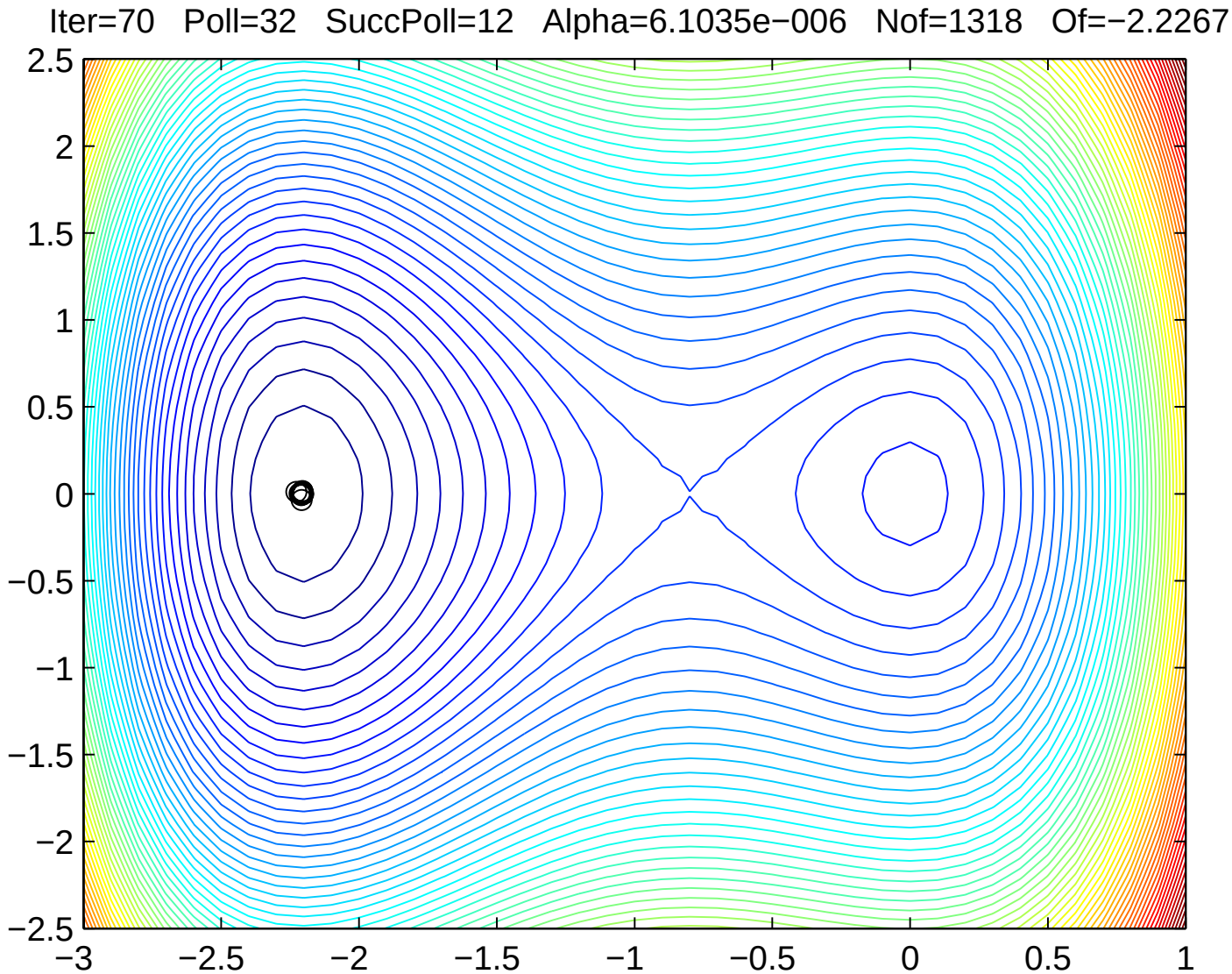

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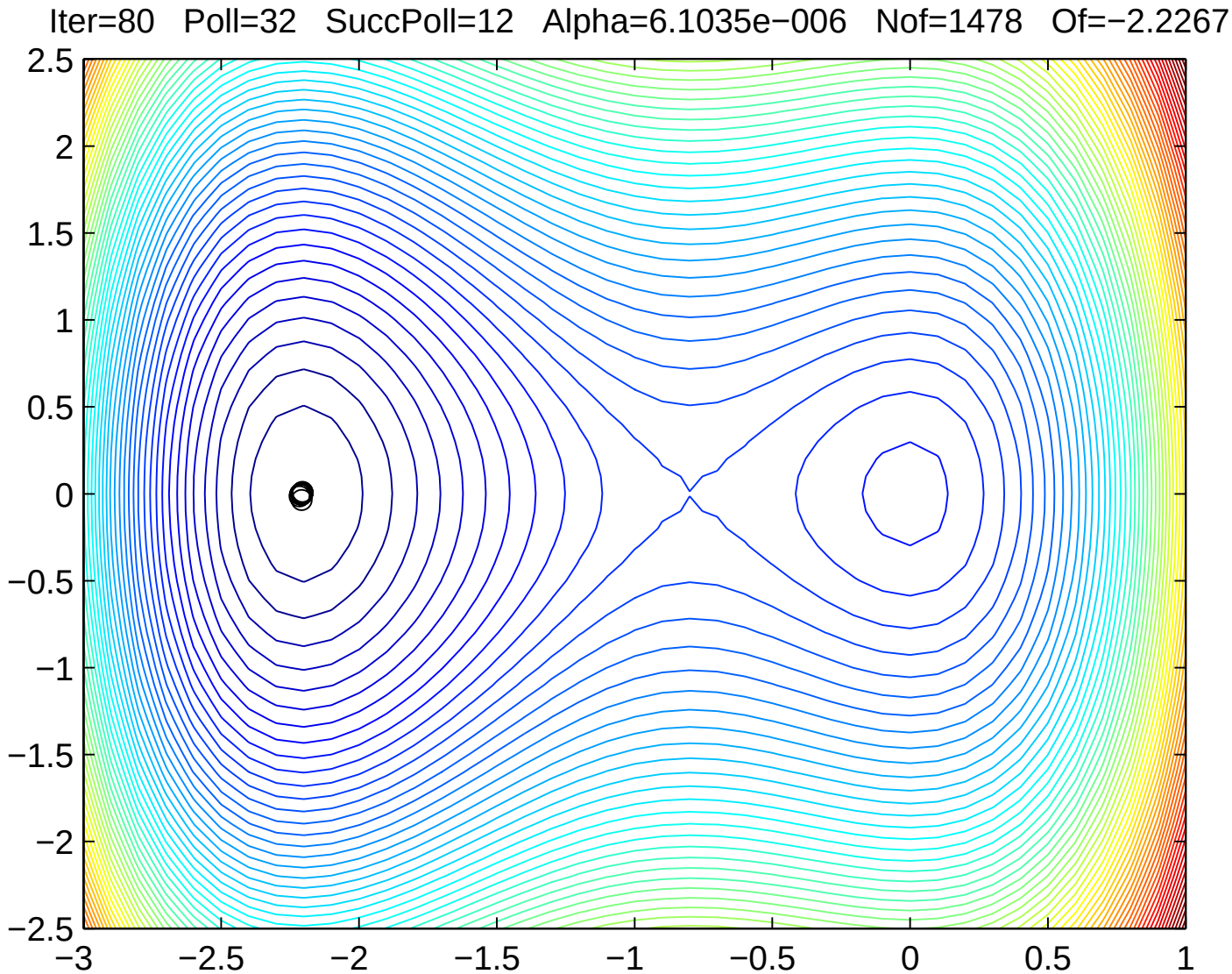

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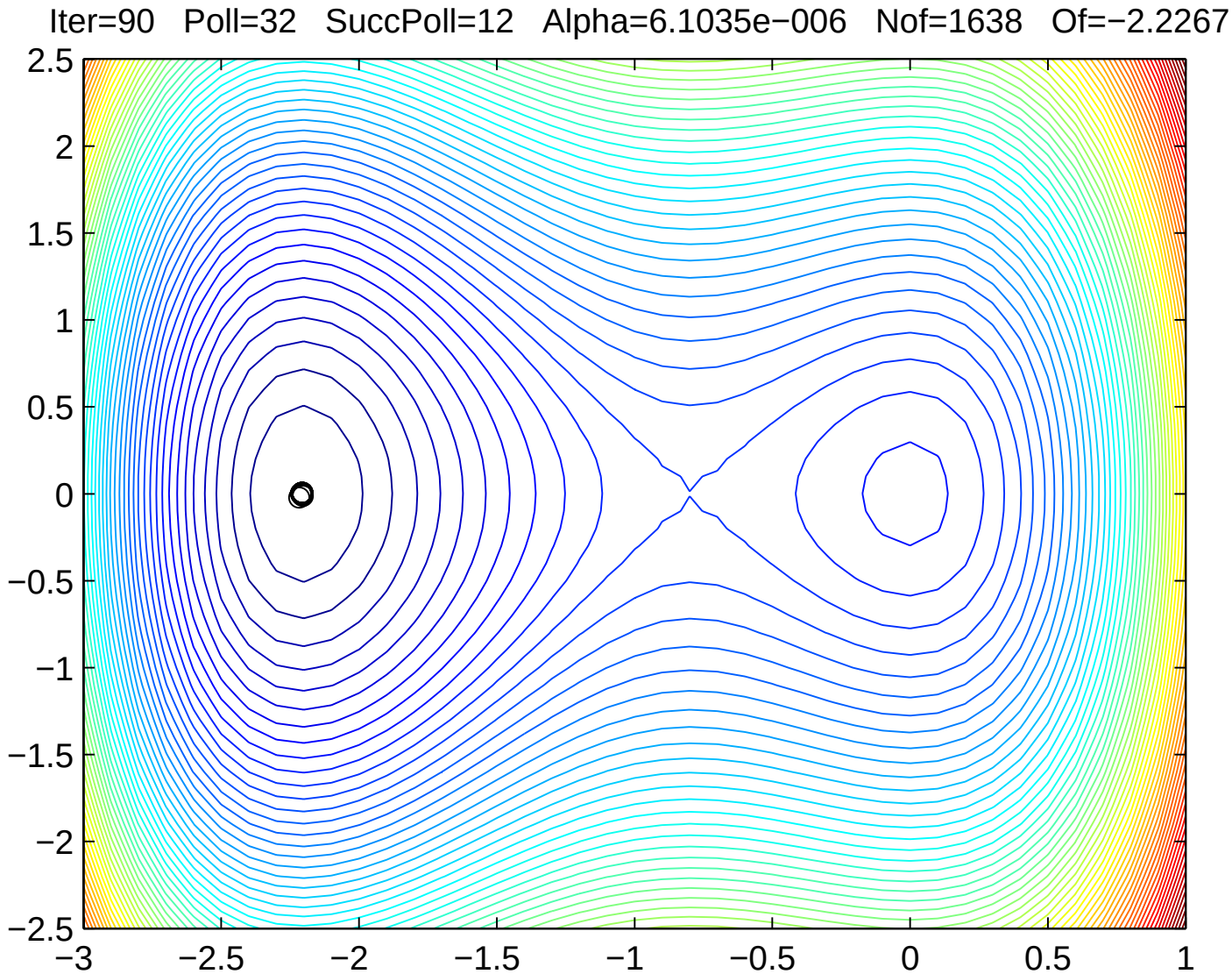

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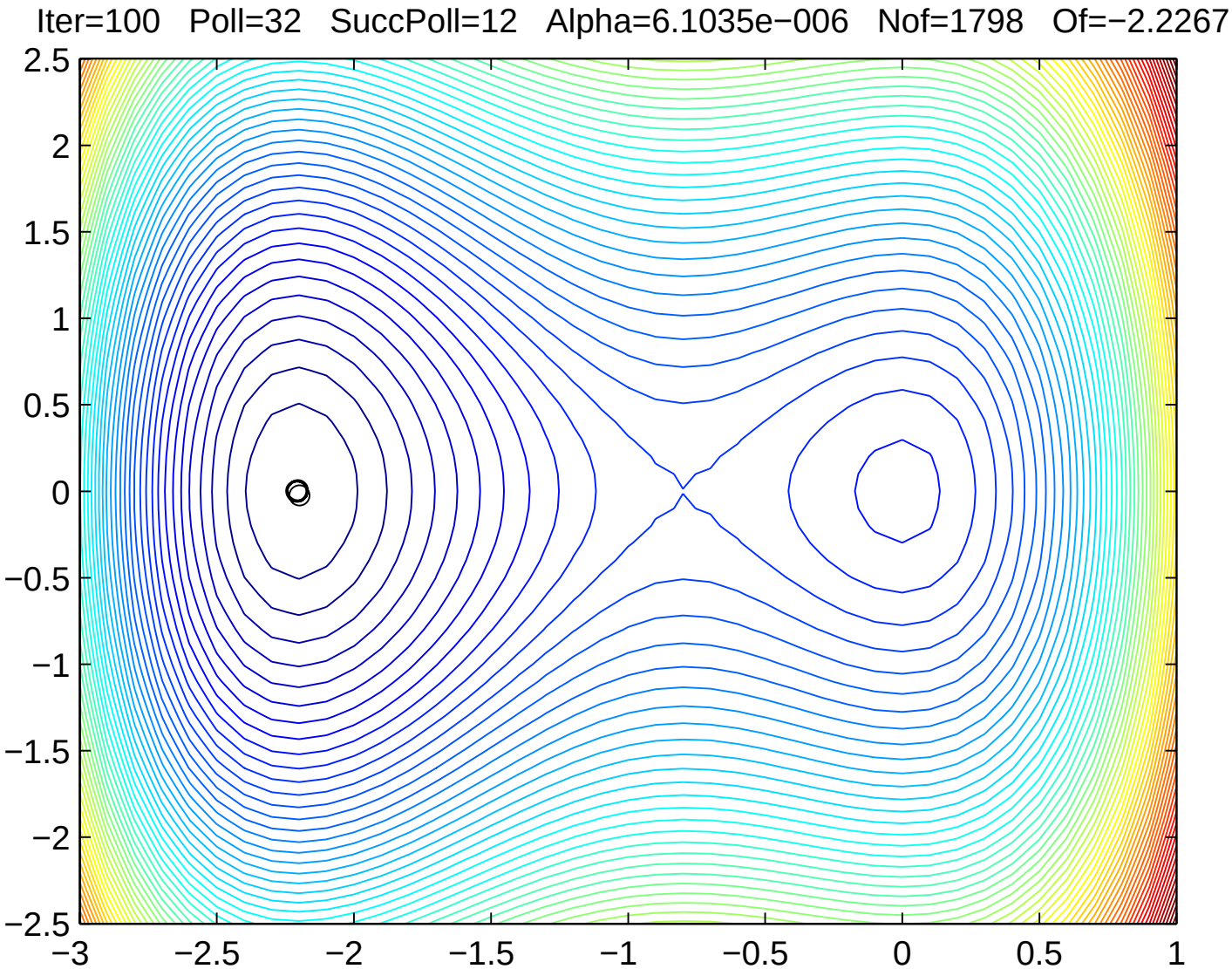

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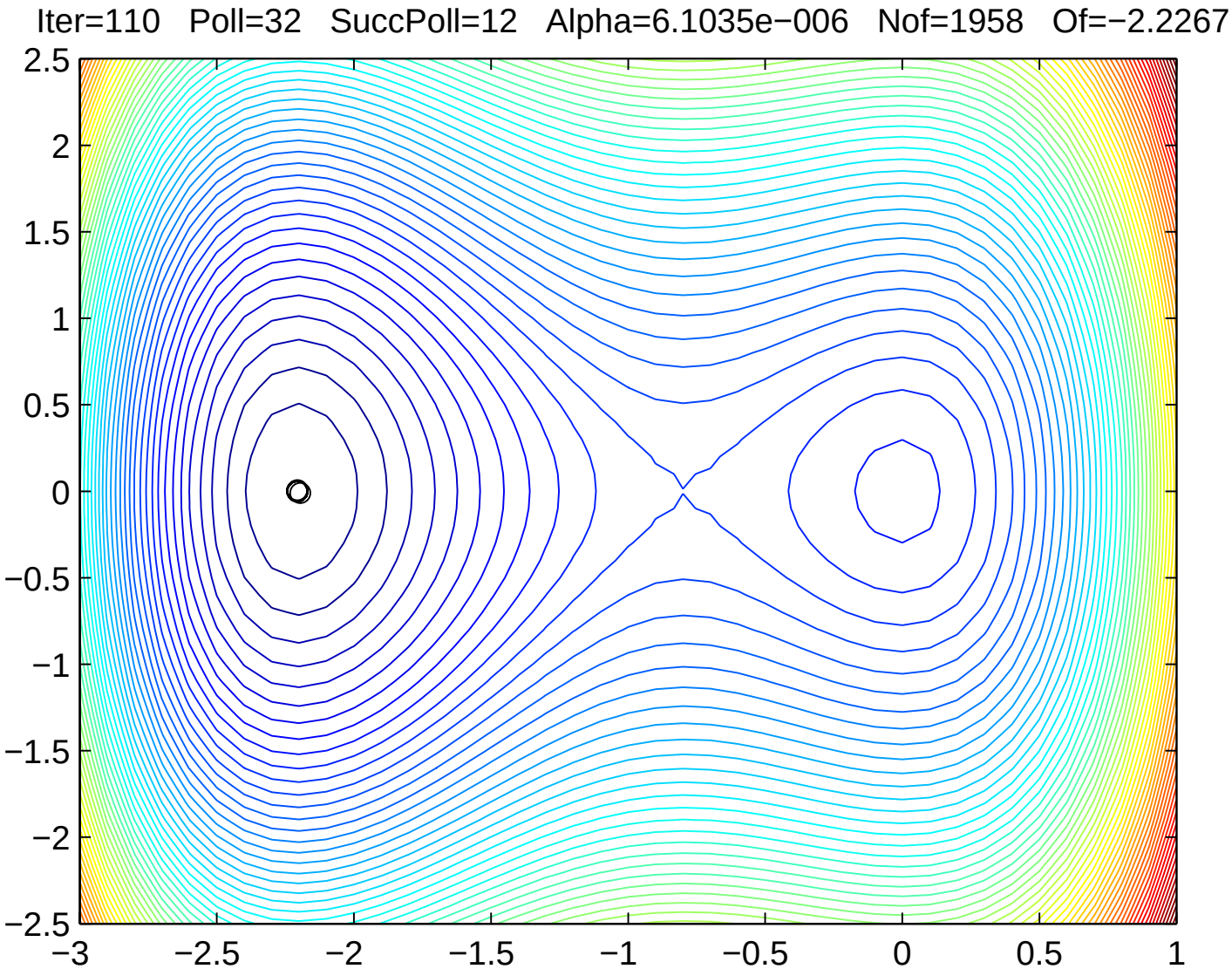

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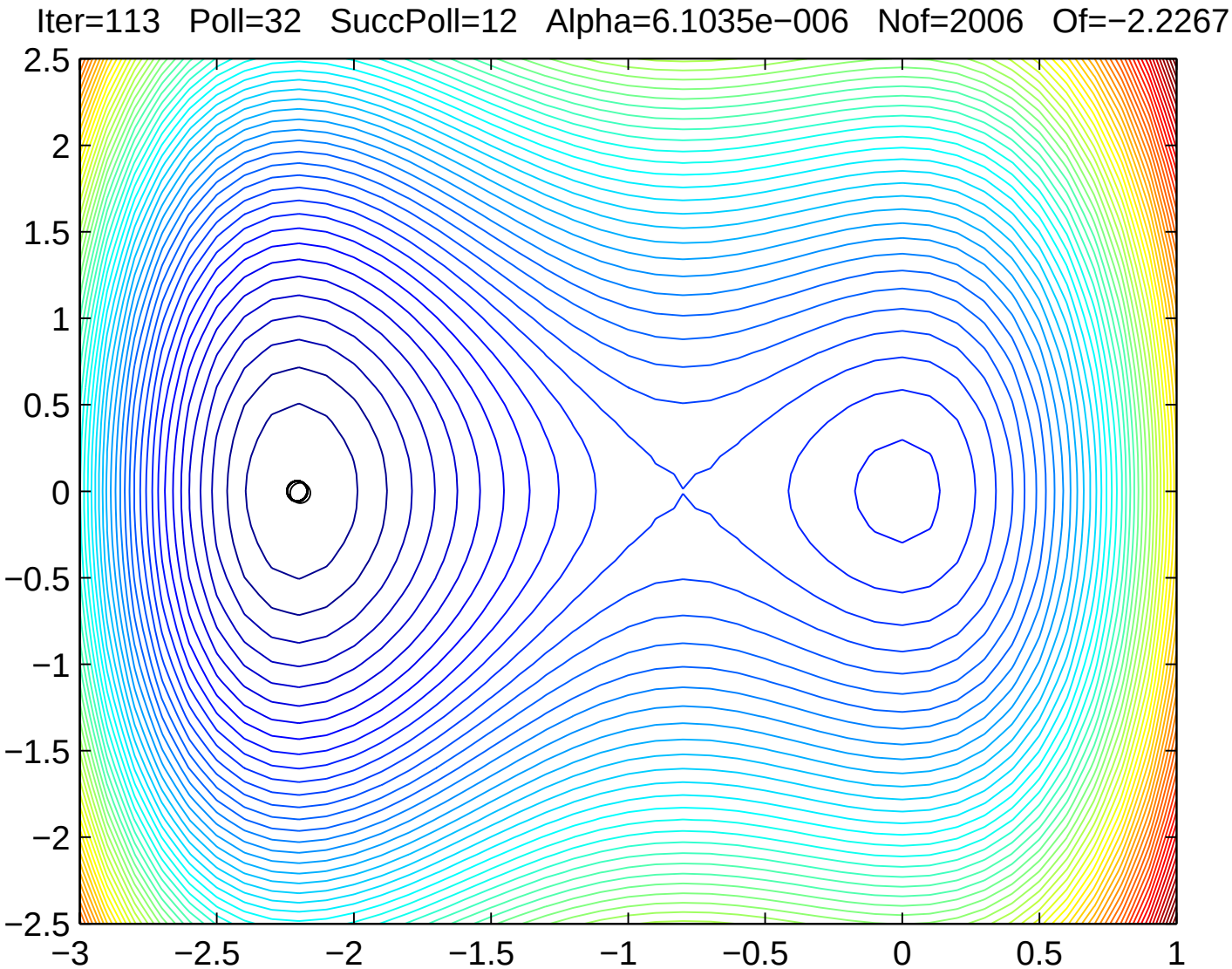
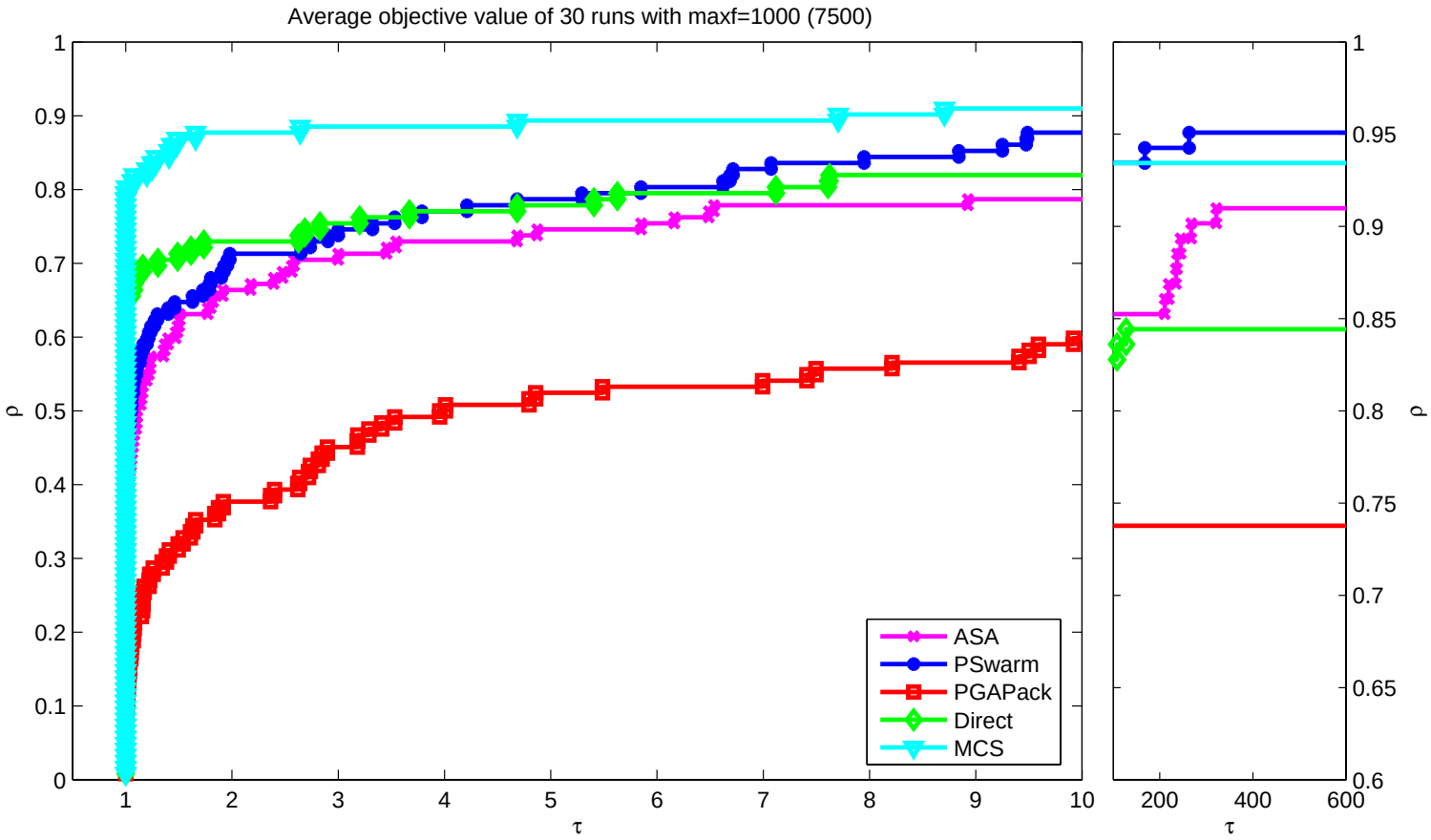


Numerical results (final value for f)

- 122 problems where 12 are of large dimension (100-300 variables).



For further details see Vaz and Vicente, JOGO, 2007

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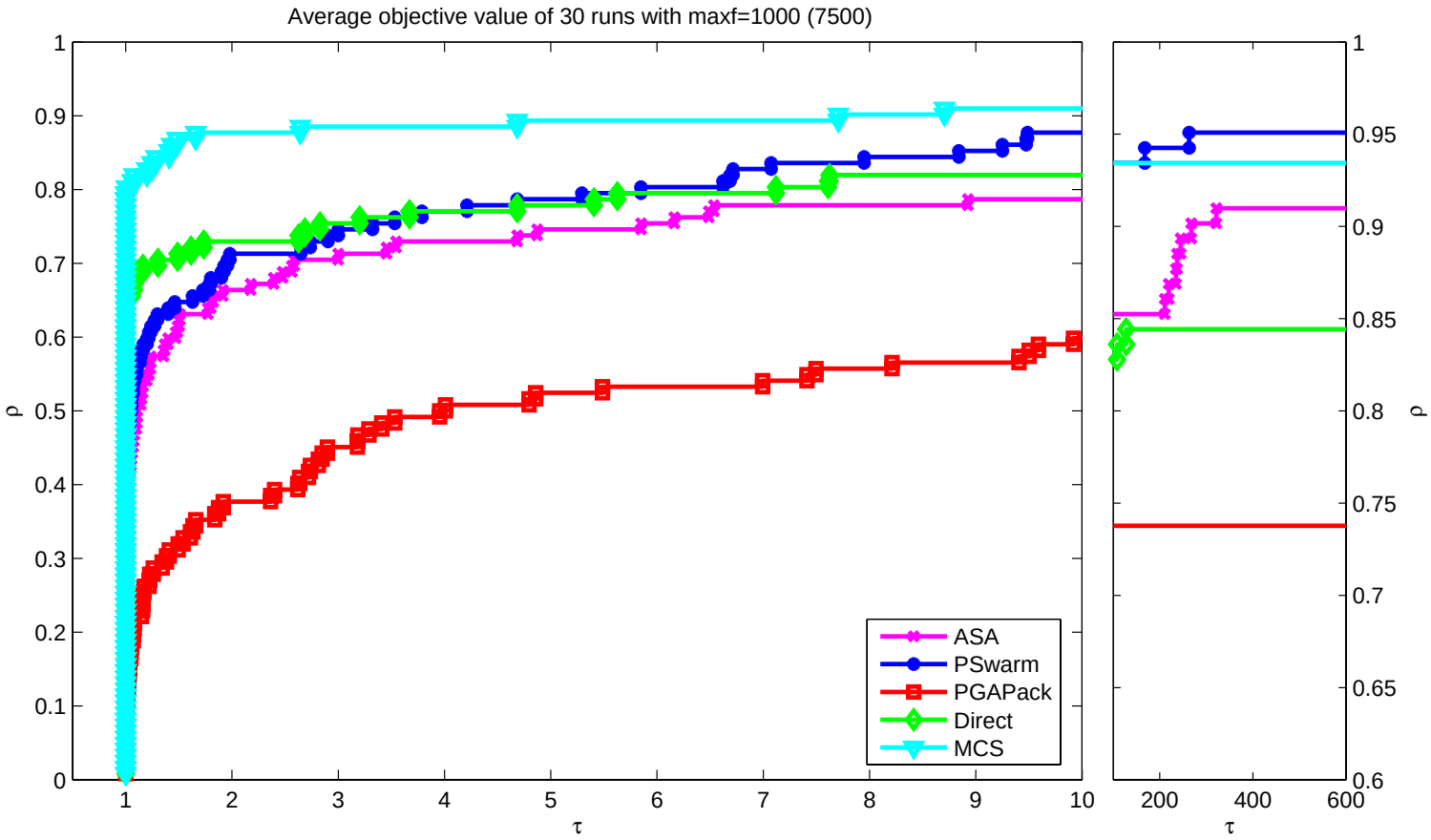

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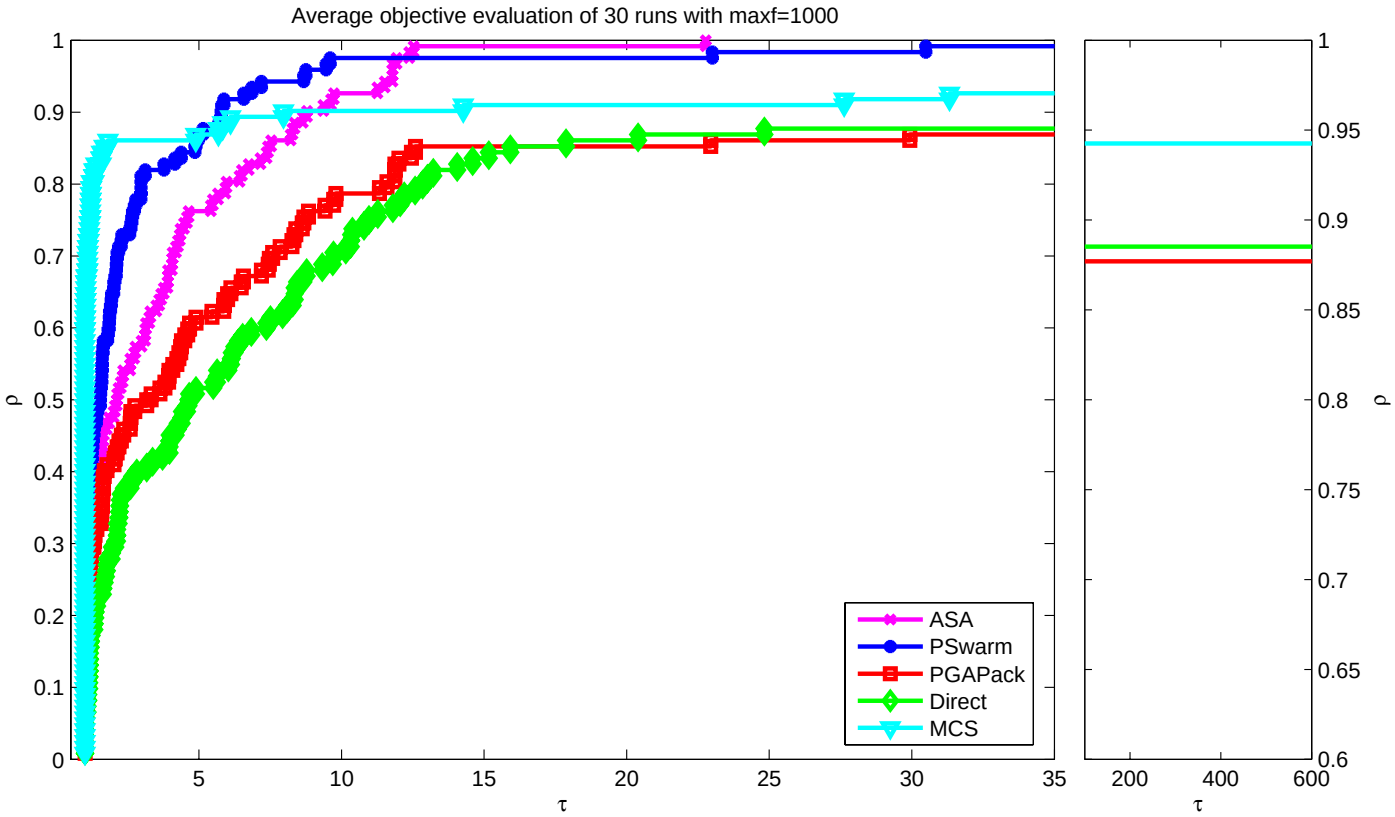
Numerical results (number of evaluations)

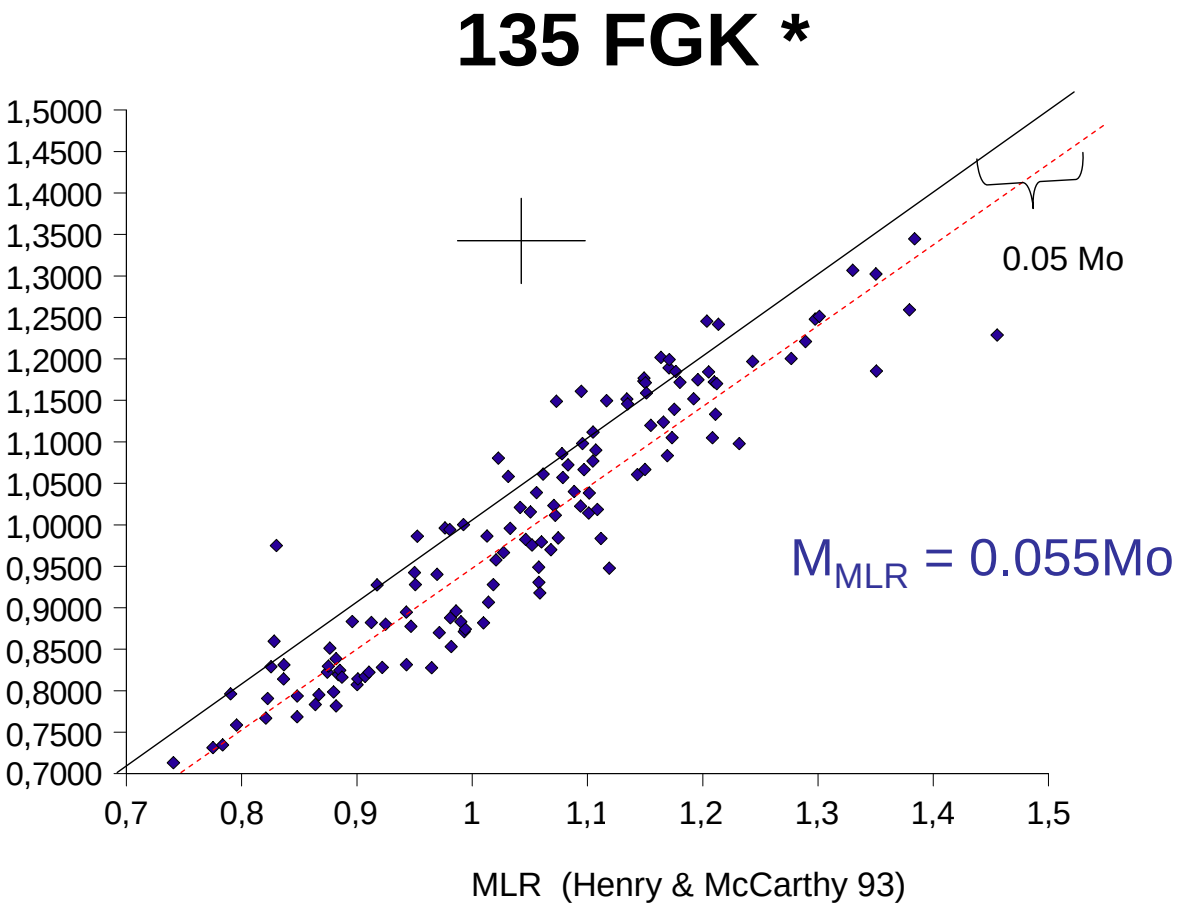


Average number of objective function evaluations.

$maxf$	ASA	PGAPack	PSwarm	Direct	MCS
1000	857	1009*	686	1107*	1837*
10000	5047	10009*	3603	11517*	4469

Parameter estimation in Astrophysics

The goal is to determine a set of six stellar parameters from observable data. The objective function requires simulation (CESAM code). PSwarm was very successful on a set of 135 stars ($193 \times 2000 \times 25 = 18.40$ years computational time in parallel).



* Stars belonging to spectral types F, G, or K.

Mass-luminosity relation (MLR)

Joint work with J.M. Fernandes - University of Coimbra.

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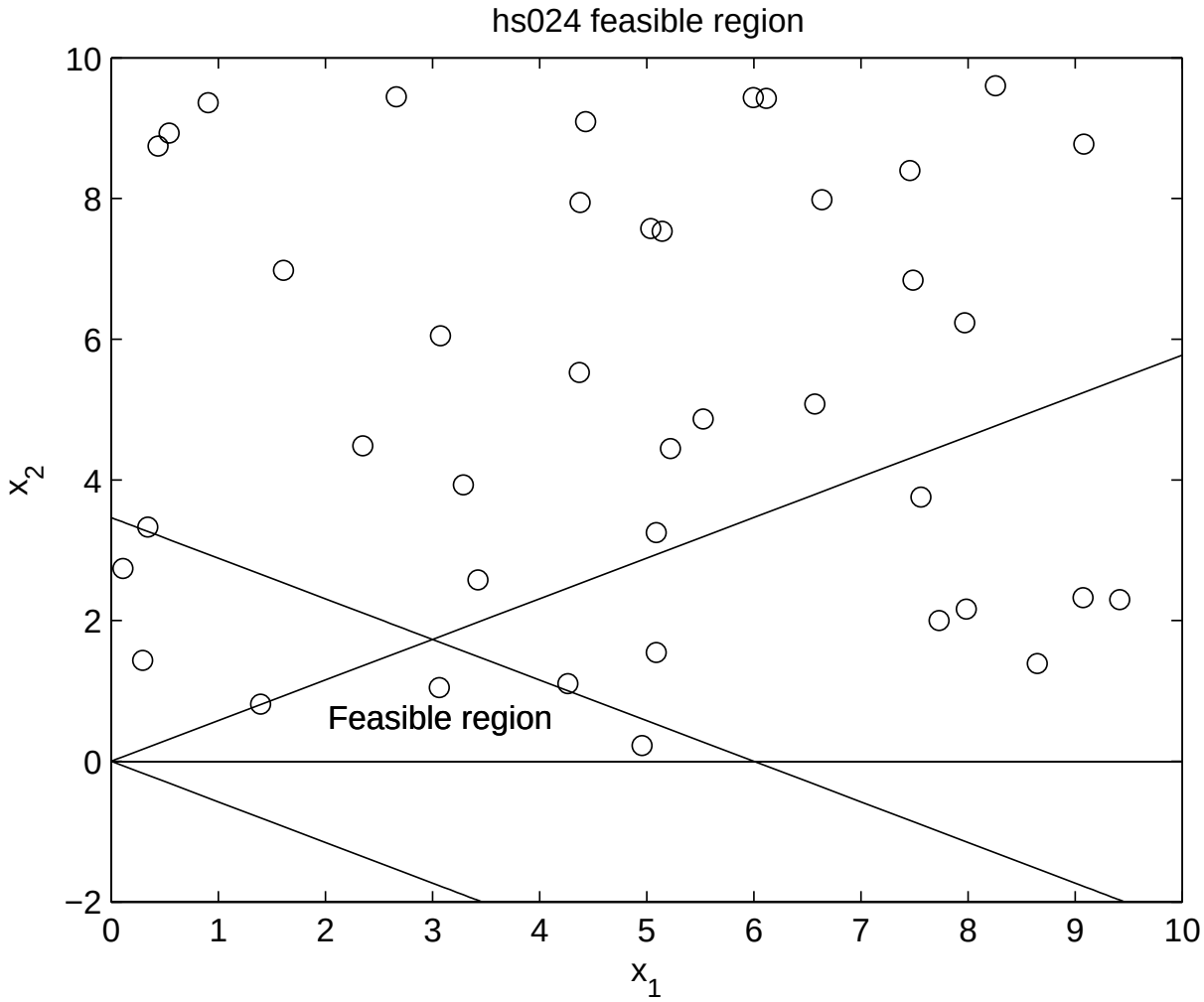
where A is a $m \times n$ matrix, b is a m column vector and $\ell \leq z \leq u$ are understood componentwise.

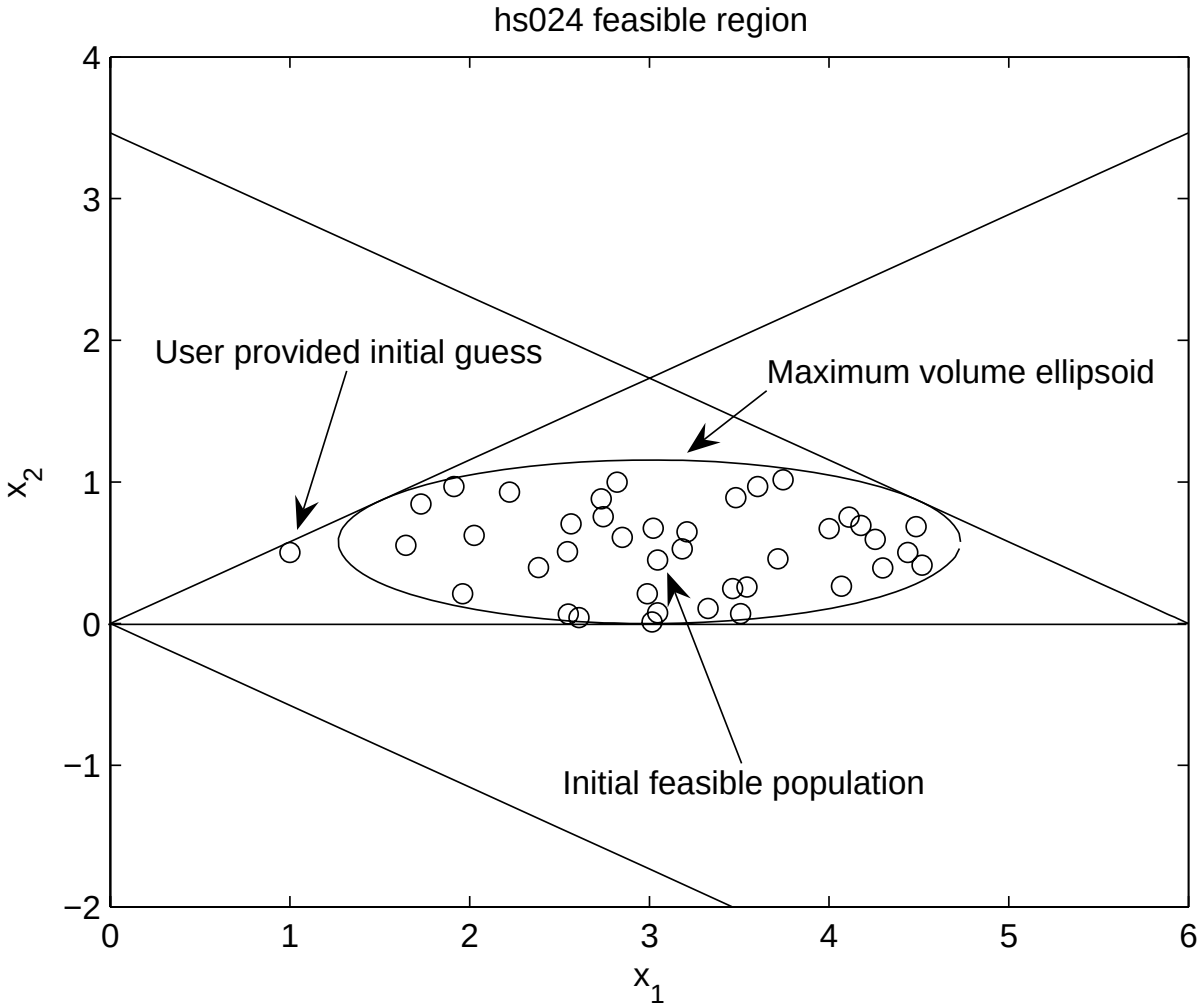
Feasible initial population

Obtaining an initial feasible population and controlling feasibility in the linear constrained case is critical.



Feasible initial population

Getting an initial feasible population allows a more efficient search for the global optimum.



Zhang and Gao interior-point code is being used to compute the maximum volume ellipsoid.

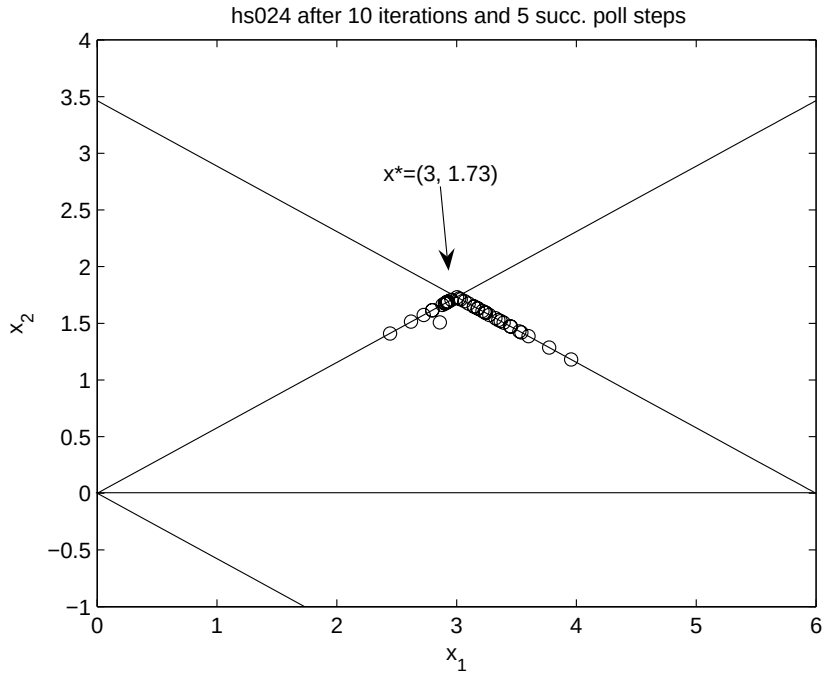
Search step (Particle Swarm)

Feasibility is kept during the optimization process for all particles. This is achieved by introducing a maximum allowed step in the "search" direction.

Maximum allowed step

$$x^p(t+1) = x^p(t) + \alpha_{max} v^p(t+1),$$

where α_{max} is the maximum step allowed to keep $x^p(t+1)$ inside the feasible region.



Poll step

For the coordinate search method applied to bound constrained problems it is sufficient to initialize the algorithm with a feasible initial guess ($y(0) \in \Omega$) and to use $\hat{f}$ as the objective function.

Penalty/Barrier function

$$\hat{f}(z) = \begin{cases} f(z) & \text{if } z \in \Omega, \\ +\infty & \text{otherwise.} \end{cases}$$

Linear constraints

For the case of linear constraints this is no longer true.

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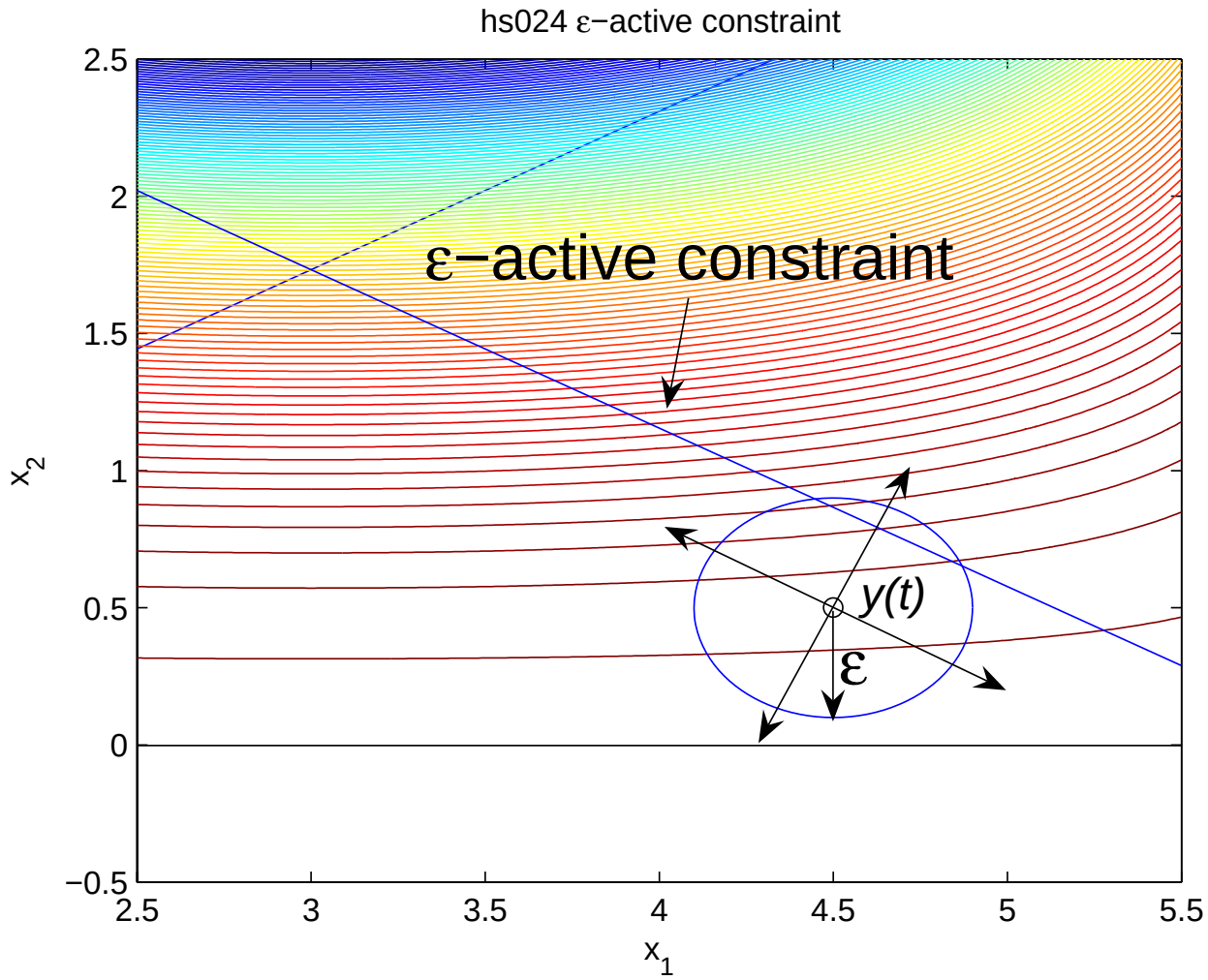
Positive generators for the tangent cone

The set of polling directions needs to conform with the geometry of the feasible set.



Positive generators for the tangent cone

No ϵ-active constraints

The positive spanning set is the maximal positive basis $D_{\oplus}$.

For ϵ-active constraint(s)

The polling directions are the positive generators for the tangent cone of the ϵ-active constraints (obtained by QR factorization)

Degeneracy

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Test problems

- 120 problems with linear constraints were collected from 1564 optimization problems (AMPL, CUTE, GAMS, NETLIB, etc.).
- 23 linear, 55 quadratic and 32 general nonlinear.
- 10 highly non-convex objective functions with random generated linear constraints (Pinter).
- The test problems are coded in AMPL (*A Modeling Language for Mathematical Programming*).
- Test problems available at `http://www.norg.uminho.pt/aivaz` (under *software*).

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- 120 problems with linear constraints were collected from 1564 optimization problems (AMPL, CUTE, GAMS, NETLIB, etc.).
- 23 linear, 55 quadratic and 32 general nonlinear.
- 10 highly non-convex objective functions with random generated linear constraints (Pinter).

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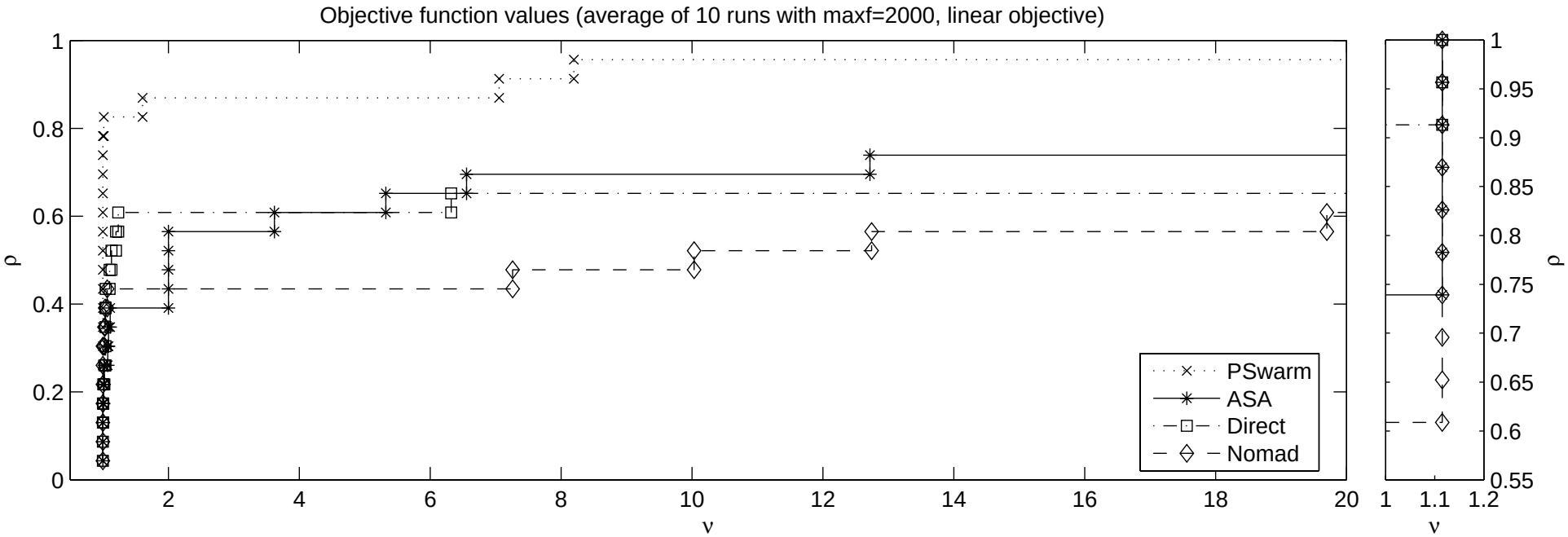
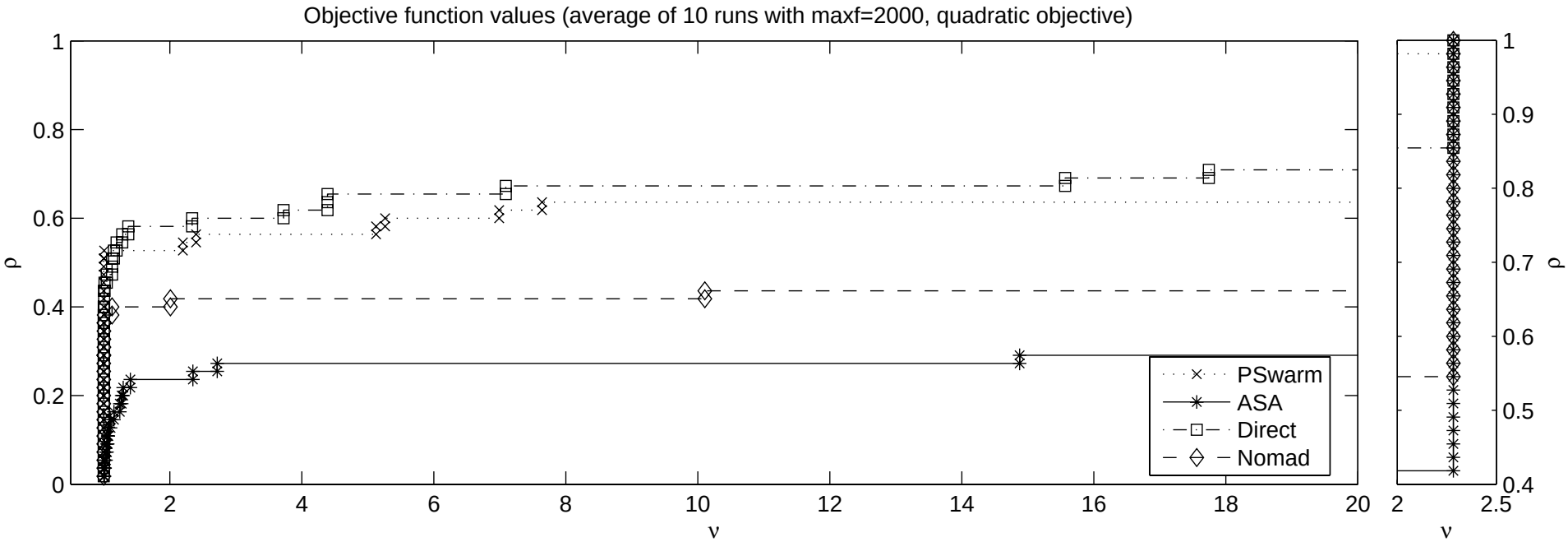
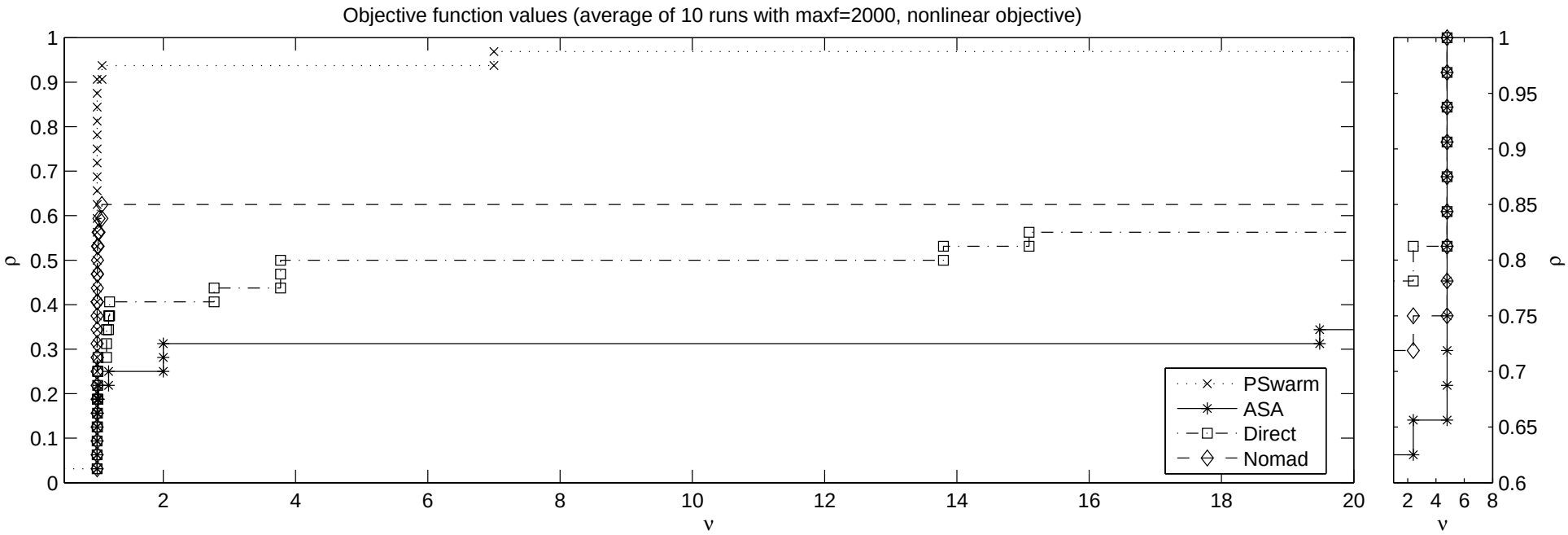
Linear objective functions



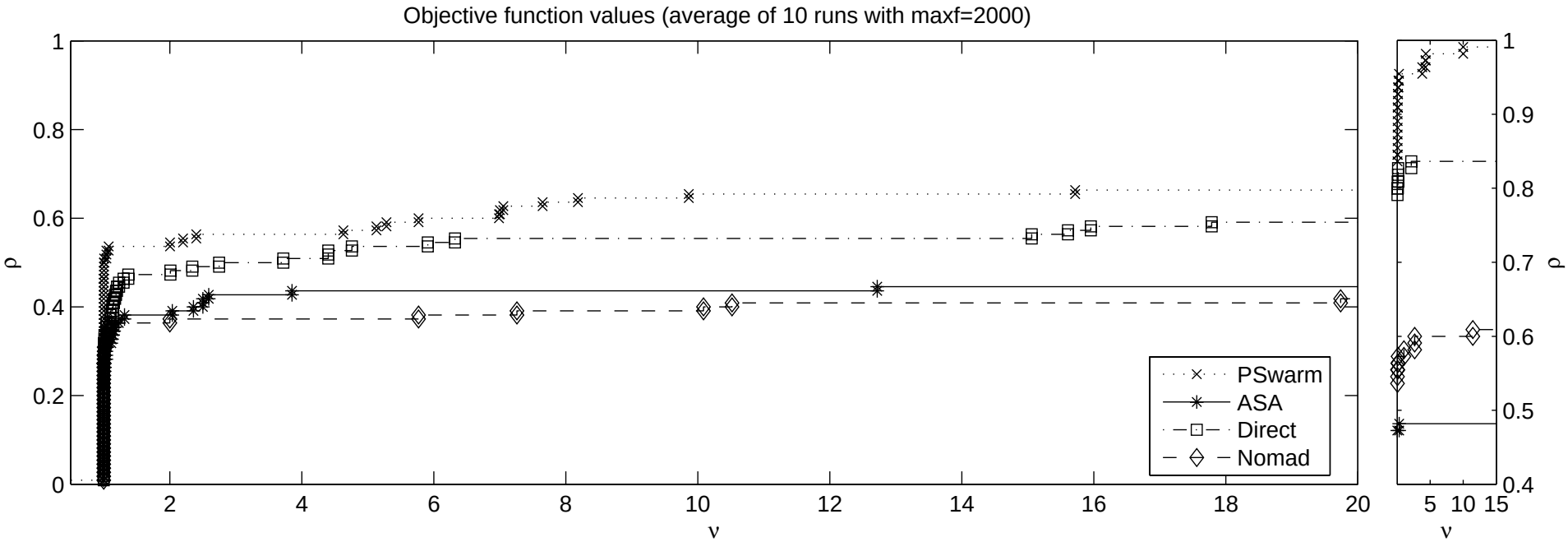
Quadratic objective functions



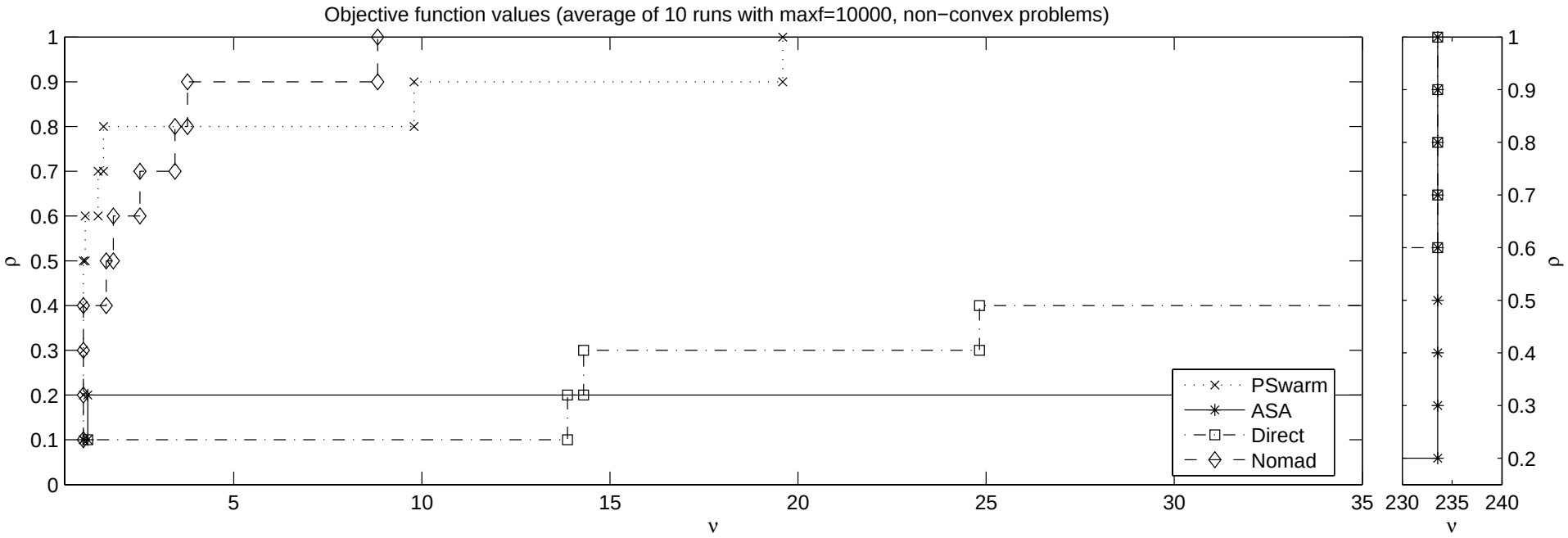
General nonlinear objective functions



All objective functions



Highly non-convex objective functions



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The end

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