Semi-infinite air pollution control problems

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Contents

- Semi-Infinite Programming (SIP)
- Dispersion model
- Formulations
- Examples
- Numerical results
Semi-infinite programming

\[
\min_{u \in \mathbb{R}^n} f(u)
\]

s.t. \( g_i(u, v) \leq 0, \ i = 1, \ldots, m \)

\[ u_{lb} \leq u \leq u_{ub} \]

\[ \forall v \in \mathcal{R} \subset \mathbb{R}^p, \]

where \( f(u) \) is the objective function, \( g_i(u, v), i = 1, \ldots, m \) are the infinite constraint functions and \( u_{lb}, u_{ub} \) are the lower and upper bounds on \( u \).
Coordinate system

(a, b) stack position

\(d\) stack internal diameter

\(h\) stack height

\(\Delta H\) plume rise

\(H = h + \Delta H\) effective stack height

\(\theta\) mean wind direction
Dispersion model

Assuming that the plume has a Gaussian distribution, the concentration, of gas or aerosol (particles with diameter less than 20 microns) at position $x$, $y$ and $z$ of a continuous source with effective stack height $H$, is given by

$$C(x, y, z, H) = \frac{Q}{2\pi \sigma_y \sigma_z U} e^{-\frac{1}{2} \left( \frac{y}{\sigma_y} \right)^2} \left( e^{-\frac{1}{2} \left( \frac{z-H}{\sigma_z} \right)^2} + e^{-\frac{1}{2} \left( \frac{z+H}{\sigma_z} \right)^2} \right)$$

where $Q \ (gs^{-1})$ is the pollution uniform emission rate, $U \ (ms^{-1})$ is the mean wind speed affecting the plume, $\sigma_y \ (m)$ and $\sigma_z \ (m)$ are the standard deviations in the horizontal and vertical planes, respectively.
Change of coordinates

The source change of coordinates to position \((a, b)\), in the wind direction. \(Y\) is given by

\[
Y = (x - a) \sin(\theta) + (y - b) \cos(\theta),
\]

where \(\theta\) (rad) is the wind direction \((0 \leq \theta \leq 2\pi)\).

\(\sigma_y\) and \(\sigma_z\) depend on \(X\) given by

\[
X = (x - a) \cos(\theta) - (y - b) \sin(\theta).
\]
Plume rise

The effective emission height is the sum of the stack height, \( h \) (m), with the plume rise, \( \Delta H \) (m). The considered elevation is given by the Holland equation

\[
\Delta H = \frac{V_0 d}{U} \left( 1.5 + 2.68 \frac{T_o - T_e}{T_o} d \right),
\]

where \( d \) (m) is the internal stack diameter, \( V_0 \) (ms\(^{-1}\)) is the gas out velocity, \( T_o \) (K) is the gas temperature and \( T_e \) (K) is the environment temperature.
Formulations

- Assuming \( n \) pollution sources distributed in a region;
Formulations

- Assuming $n$ pollution sources distributed in a region;
- $C_i$ is the source $i$ contribution for the total concentration;
Formulations

- Assuming $n$ pollution sources distributed in a region;
- $C_i$ is the source $i$ contribution for the total concentration;
- Gas chemical inert.
Formulations

- Assuming $n$ pollution sources distributed in a region;
- $C_i$ is the source $i$ contribution for the total concentration;
- Gas chemical inert.

We can derive three formulations:

- Minimize the stack height;
- Maximum pollution computation and sampling stations planning;
- Air pollution abatement.
Minimum stack height

Minimizing the stack height \( u = (h_1, \ldots, h_n) \), while the pollution ground pollution level is kept below a given threshold \( C_0 \), in a given region \( \mathcal{R} \), can be formulated as a SIP problem

\[
\min_{u \in \mathbb{R}^n} \sum_{i=1}^{n} c_i h_i \\
\text{s.t. } g(u, v \equiv (x, y)) = \sum_{i=1}^{n} C_i(x, y, 0, H_i) \leq C_0 \\
\forall v \in \mathcal{R} \subset \mathbb{R}^2,
\]

where \( c_i, i = 1, \ldots, n \), are the construction costs.

Note: more complex objective function can be considered.
Maximum pollution and sampling stations planning

The maximum pollution concentration ($l^*$) in a given region can be obtained by solving the following SIP problem

$$\min_{l \in \mathbb{R}} l$$

s.t. $g(z, v \equiv (x, y)) \equiv \sum_{i=1}^{n} C_i(x, y, 0, H_i) \leq l$

$\forall v \in \mathcal{R} \subset \mathbb{R}^2$.

The active points $v^* \in \mathcal{R}$ where $g(z^*, v^*) = l^*$ are the global optima and indicate where the sampling (control) stations should be placed.
Air pollution abatement

Minimizing the pollution abatement (minimizing clean costs, maximizing the revenue, minimizing the economical impact) while the air pollution concentration is kept below a given threshold can be posed as a SIP problem

$$\min_{u \in \mathbb{R}^n} \sum_{i=1}^{n} p_i r_i$$

s.t. \( g(u, v \equiv (x, y)) \equiv \sum_{i=1}^{n} (1 - r_i) C_i(x, y, 0, \mathcal{H}_i) \leq C_0 \)

\( \forall v \in \mathcal{R} \subset \mathbb{R}^2 \),

where \( u = (r_1, \ldots, r_n) \) is the pollution reduction and \( p_i, i = 1, \ldots, n \), is the source \( i \) cost (cleaning or not producing).
Example - Minimum stack height

Consider a region with 10 stacks. The environment temperature ($T_e$) is $283 \, K$ and the emission gas temperature ($T_o$) is $413 \, K$. The wind velocity ($U$) is $5.64 \, ms^{-1}$ in the $3.996\, rad$ direction ($\theta$).

The stack height in the table were used as initial guess and a squared region of $40\, km$ was considered ($\mathcal{R} = [-20000, 20000] \times [-20000, 20000]$).
Data for the 10 stacks

The stacks data is

<table>
<thead>
<tr>
<th>Source</th>
<th>$a_i$ (m)</th>
<th>$b_i$ (m)</th>
<th>$h_i$ (m)</th>
<th>$d_i$ (m)</th>
<th>$Q_i$ (gs$^{-1}$)</th>
<th>$(V_o)_i$ (ms$^{-1}$)</th>
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<tr>
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<td>-1700</td>
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<td>2391.3</td>
<td>17.690</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>-2500</td>
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<td>7.6</td>
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<td>6.3</td>
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<td>23.404</td>
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<td>91.4</td>
<td>5.0</td>
<td>1304.3</td>
<td>22.293</td>
</tr>
</tbody>
</table>
Numerical results

Two threshold values were tested. $C_0 = 7.7114 \times 10^{-4} \text{gm}^{-3}$ without a lower bound on the stack height, $C_0 = 7.7114 \times 10^{-4} \text{gm}^{-3}$ with a stack lower bound height of 10m$^1$ and $C_0^2 = 1.25 \times 10^{-4} \text{gm}^{-3}$.

The stack height can only be inferior to 10m if some legal$^3$ requirements are met. One way to prove that the requirements are met is by simulation, using a proper model, of the air pollution dispersion.

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$^1$Decree law number 352/90 from 9 November 1990.
$^2$Decree law number 111/2002 from 16 April 2002.
$^3$Decree law number 286/93 from 12 March 1993.
# Numerical results

<table>
<thead>
<tr>
<th></th>
<th>Instance 1</th>
<th>Instance 2</th>
<th>Instance 3</th>
</tr>
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<td>Total</td>
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Constraint contour
Example - Maximum pollution level and sampling stations planning

Computing the maximum pollution level ($l^*$) by fixing the stack height $h_i$.

The region considered was $\mathcal{R} = [0, 24140] \times [0, 24140]$ (square of about 15 miles).

Environment temperature of $284K$, and wind velocity of $5m/s^{-1}$ in direction $3.927rad$ ($225^\circ$).
## Data for the 25 stacks

<table>
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<th>Source</th>
<th>$a_i$ (m)</th>
<th>$b_i$ (m)</th>
<th>$h_i$ (m)</th>
<th>$d_i$ (m)</th>
<th>$Q_i$ (gs$^{-1}$)</th>
<th>$(V_0)_i$ (ms$^{-1}$)</th>
<th>$(T_0)_i$ (K)</th>
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<td>6.1</td>
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<td>171.3</td>
<td>16.1</td>
<td>616</td>
</tr>
</tbody>
</table>
Numerical results - contour

The maximum pollution level of \( l^* = 1.81068 \times 10^{-3} \text{ gm}^{-3} \) in position \((x, y) = (8500, 7000)\).
Example - Air pollution abatement

Consider three plants ($P_1$, $P_2$ and $P_3$), with emissions of $e_1$, $e_2$ and $e_3$, where $0 \leq e_i \leq 2$, ($i = 1, 2, 3$) of a certain pollutant. By legal imposition the pollution level must not exceed a given threshold ($C_0$) under mean weather conditions, i.e., $\theta = 0$ and $U = \left(\frac{1}{2\pi}\right)^2 ms^{-1}$. Consider $Q = 1 gs^{-1}$ and $C_0 = \frac{1}{2}$. The remaining stacks data are

<table>
<thead>
<tr>
<th>Source</th>
<th>$a_i$</th>
<th>$b_i$</th>
<th>$h_i$</th>
</tr>
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<tbody>
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<td>1</td>
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<td>1</td>
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<tr>
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<tr>
<td>3</td>
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<td>$\sqrt{2}$</td>
</tr>
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</table>
**Problem**

The emission rate reduction is to be minimized.

\[
\min_{r_1, r_2, r_3 \in \mathbb{R}} \quad 2r_1 + 4r_2 + r_3
\]

s.t. \( \sum_{i=1}^{3} (2 - r_i) C(x, y, 0, \mathcal{H}_i) \leq C_0 \)

0 \leq r_i \leq 2, \ i = 1, 2, 3

\( \forall (x, y) \in [-1, 4] \times [-1, 4] \).
Numerical results

Solution found $r^* = (0.987, 0.951, 0.943)$

The maximum pollution is attained at $(x, y)^1 = (1.100, 0.125)$, $(x, y)^2 = (1.100, 0.100)$ and $(x, y)^3 = (3.675, -0.625)$, where the sampling stations should be placed.
Constraint contour
Conclusions

- Air pollution control problems formulated as SIP problems;
Conclusions

- Air pollution control problems formulated as SIP problems;

- Problems coded in (SIP)AMPL modeling language.
  
  vaz1.mod  Minimum stack height
  vaz2.mod  Maximum attained pollution and sampling stations planning
  vaz3.mod  Air pollution abatement

Publicly available together with the SIPAMPL at http://www.norg.uminho.pt/aivaz/;
Conclusions

- Air pollution control problems formulated as SIP problems;
- Problems coded in (SIP)AMPL modeling language.
  - `vaz1.mod` Minimum stack height
  - `vaz2.mod` Maximum attained pollution and sampling stations planning
  - `vaz3.mod` Air pollution abatement

Publicly available together with the SIPAMPL at

http://www.norg.uminho.pt/aivaz/

- Numerical results obtained with the NSIPS solver;
The End

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ecferreira@deb.uminho.pt
Web http://www.norg.uminho.pt/aivaz/