Air pollution control with semi-infinite programming

A. Ismael F. Vaz
Production and Systems Department
aivaz@dps.uminho.pt

Eugénio C. Ferreira
Biological Engineering Center
ecferreira@deb.uminho.pt

Engineering School

Minho University - Braga - Portugal

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Contents

Semi-infinite programming

$$\min_{u \in R^n} f(u)$$

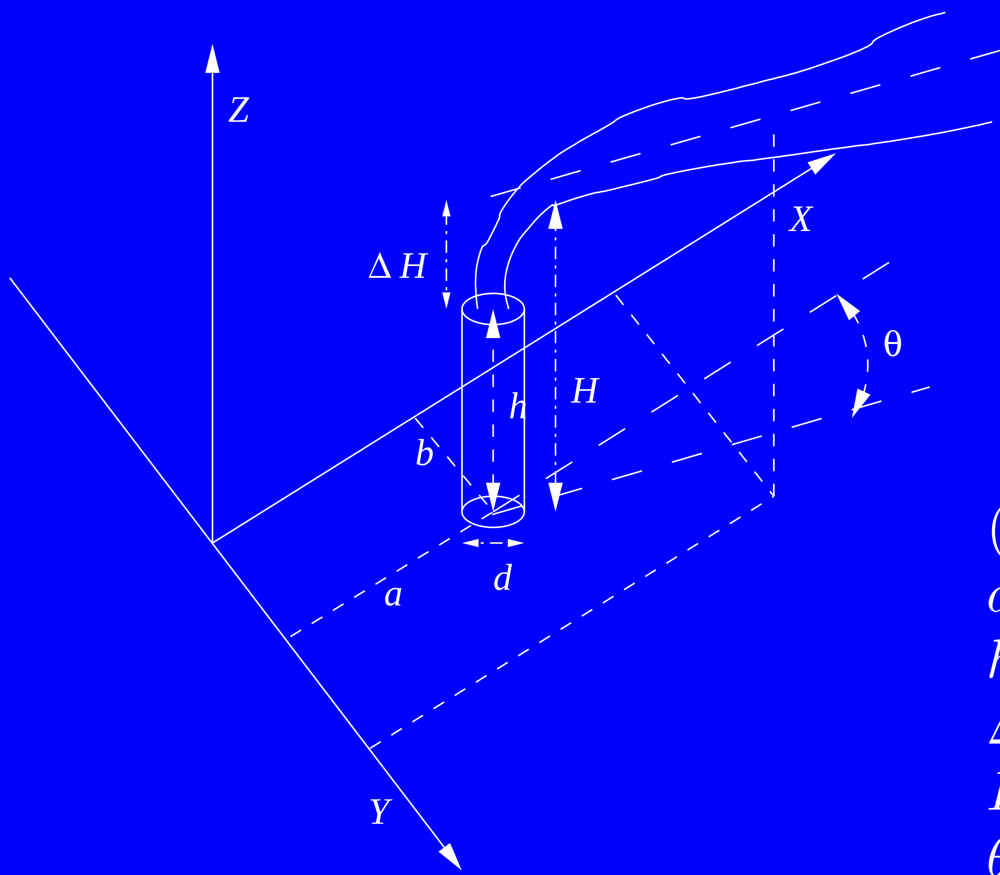
$$s.t. \;\; g_i(u,v) \leq 0, \; i = 1, \ldots, m$$

$$u_{lb} \leq u \leq u_{ub}$$

$$\forall v \in \mathcal{R} \subset R^p,$$

where $f(u)$ is the objective function, $g_i(u,v)$, $i = 1, \ldots, m$ are the infinite constraint functions and u_{lb}, u_{ub} are the lower and upper bounds on u.

Coordinate system



(a, b)	stack position
d	stack internal diameter
h	stack height
ΔH	plume rise
$H = h + \Delta H$	effective stack height
θ	mean wind direction

Dispersion model

Assuming that the plume has a Gaussian distribution, the concentration, of gas or aerosol (particles with diameter less than 20 microns) at position x, y and z of a continuous source with effective stack height $\mathcal{H}$, is given by

$$\mathcal{C}(x,y,z,\mathcal{H}) = \frac{\mathcal{Q}}{2\pi\sigma_y\sigma_z\mathcal{U}} e^{-\frac{1}{2}\left(\frac{\mathcal{Y}}{\sigma_y}\right)^2} \left(e^{-\frac{1}{2}\left(\frac{z-\mathcal{H}}{\sigma_z}\right)^2} + e^{-\frac{1}{2}\left(\frac{z+\mathcal{H}}{\sigma_z}\right)^2} \right)$$

where $\mathcal{Q}$ (gs^{-1}) is the pollution uniform emission rate, $\mathcal{U}$ (ms^{-1}) is the mean wind speed affecting the plume, σ_y (m) and σ_z (m) are the standard deviations in the horizontal and vertical planes, respectively.

Change of coordinates

The source change of coordinates to position (a, b), in the wind direction.

$\mathcal{Y}$ is given by

$$\mathcal{Y} = (x - a)\sin(\theta) + (y - b)\cos(\theta),$$

where θ (rad) is the wind direction $(0 \leq \theta \leq 2\pi)$.

σ_y and σ_z depend on $\mathcal{X}$ given by

$$\mathcal{X} = (x - a)\cos(\theta) - (y - b)\sin(\theta).$$

Plume rise

The effective emission height is the sum of the stack height, h (m), with the plume rise, $\Delta\mathcal{H}$ (m). The considered elevation is given by the Holland equation

$$\Delta\mathcal{H} = \frac{V_o d}{\mathcal{U}}\left(1.5 + 2.68\frac{T_o - T_e}{T_o}d\right),$$

where d (m) is the internal stack diameter, V_o (ms^{-1}) is the gas out velocity, T_o (K) is the gas temperature and T_e (K) is the environment temperature.

Formulations

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- Gas chemical inert.

We can derive three formulations:

- Minimize the stack height;
- Maximum pollution computation and sampling stations planning;
- Air pollution abatement.

Minimum stack height

Minimizing the stack height $u = (h_1, \ldots, h_n)$, while the pollution ground pollution level is kept below a given threshold $\mathcal{C}_0$, in a given region $\mathcal{R}$, can be formulated as a SIP problem

$$\min_{u \in R^n} \sum_{i=1}^{n} c_i h_i$$

$$s.t. \;\; g(u, v \equiv (x, y)) \equiv \sum_{i=1}^{n} \mathcal{C}_i(x, y, 0, \mathcal{H}_i) \leq \mathcal{C}_0$$

$$\forall v \in \mathcal{R} \subset R^2,$$

where c_i, $i = 1, \ldots, n$, are the construction costs.

Note: more complex objective function can be considered.

Maximum pollution and sampling stations planning

The maximum pollution concentration (l^*) in a given region can be obtained by solving the following SIP problem

$$\min_{l \in R} \; l$$

$$s.t. \;\; g(z, v \equiv (x, y)) \equiv \sum_{i=1}^{n} \mathcal{C}_i(x, y, 0, \mathcal{H}_i) \leq l$$

$$\forall v \in \mathcal{R} \subset R^2.$$

The active points $v^* \in \mathcal{R}$ where $g(z^*, v^*) = l^*$ are the global optima and indicate where the sampling (control) stations should be placed.

Air pollution abatement

Minimizing the pollution abatement (minimizing clean costs, maximizing the revenue, minimizing the economical impact) while the air pollution concentration is kept below a given threshold can be posed as a SIP problem

$$\min_{u \in R^n} \sum_{i=1}^{n} p_i r_i$$

$$s.t. \;\; g(u, v \equiv (x,y)) \equiv \sum_{i=1}^{n} (1 - r_i)\mathcal{C}_i(x, y, 0, \mathcal{H}_i) \leq \mathcal{C}_0$$

$$\forall v \in \mathcal{R} \subset R^2,$$

where $u = (r_1, \ldots, r_n)$ is the pollution reduction and p_i, $i = 1, \ldots, n$, is the source i cost (cleaning or not producing).

Modeling environment

SIPAMPL stands for "Semi-Infinite Programming with AMPL". SIPAMPL extends AMPL, allowing the SIP problems to be coded and provides:

- a database with more than 160 coded problems;
- an interface between SIPAMPL and any solver (NSIPS);
- an interface between SIPAMPL and MATLAB;
- a *select* tool.

SIPAMPL was used to code the proposed examples.

Modeling and solving environment

NSIPS stands for "Nonlinear Semi-Infinite Programming Solver". NSIPS implements four different algorithms for SIP:

- Discretization;
- SQP;
- Penalty;
- Interior point.

NSIPS was used for solving the proposed problems. Discretization methods is the only one allowing finite constraints.

Example - Minimum stack height (Wang and Luus, 1978)

Consider a region with 10 stacks. The environment temperature (T_e) is $283K$ and the emission gas temperature (T_o) is $413K$. The wind velocity ($\mathcal{U}$) is $5.64ms^{-1}$ in the $3.996rad$ direction (θ).

The stack height in the table were used as initial guess and a squared region of 40km was considered ($\mathcal{R} = [-20000, 20000] \times [-20000, 20000]$).

Data for the 10 stacks

The stacks data is

Source	a_i (m)	b_i (m)	h_i (m)	d_i (m)	$\mathcal{Q}_i$ (gs^{-1})	$(V_o)_i$ (ms^{-1})
1	-3000	-2500	183	8.0	2882.6	19.245
2	-2600	-300	183	8.0	2882.6	19.245
3	-1100	-1700	160	7.6	2391.3	17.690
4	1000	-2500	160	7.6	2391.3	17.690
5	1000	2200	152.4	6.3	2173.9	23.404
6	2700	1000	152.4	6.3	2173.9	23.404
7	3000	-1600	121.9	4.3	1173.9	27.128
8	-2000	2500	121.9	4.3	1173.9	27.128
9	0	0	91.4	5.0	1304.3	22.293
10	1500	-1600	91.4	5.0	1304.3	22.293

Numerical results

Two threshold values were tested. $\mathcal{C}_0 = 7.7114 \times 10^{-4} gm^{-3}$ without a lower bound on the stack height, $\mathcal{C}_0 = 7.7114 \times 10^{-4} gm^{-3}$ with a stack lower bound height of $10m$[1] and $\mathcal{C}_0$[2] $= 1.25 \times 10^{-4} gm^{-3}$.

The stack height can only be inferior to $10m$ if some legal[3] requirements are met. One way to prove that the requirements are met is by simulation, using a proper model, of the air pollution dispersion.

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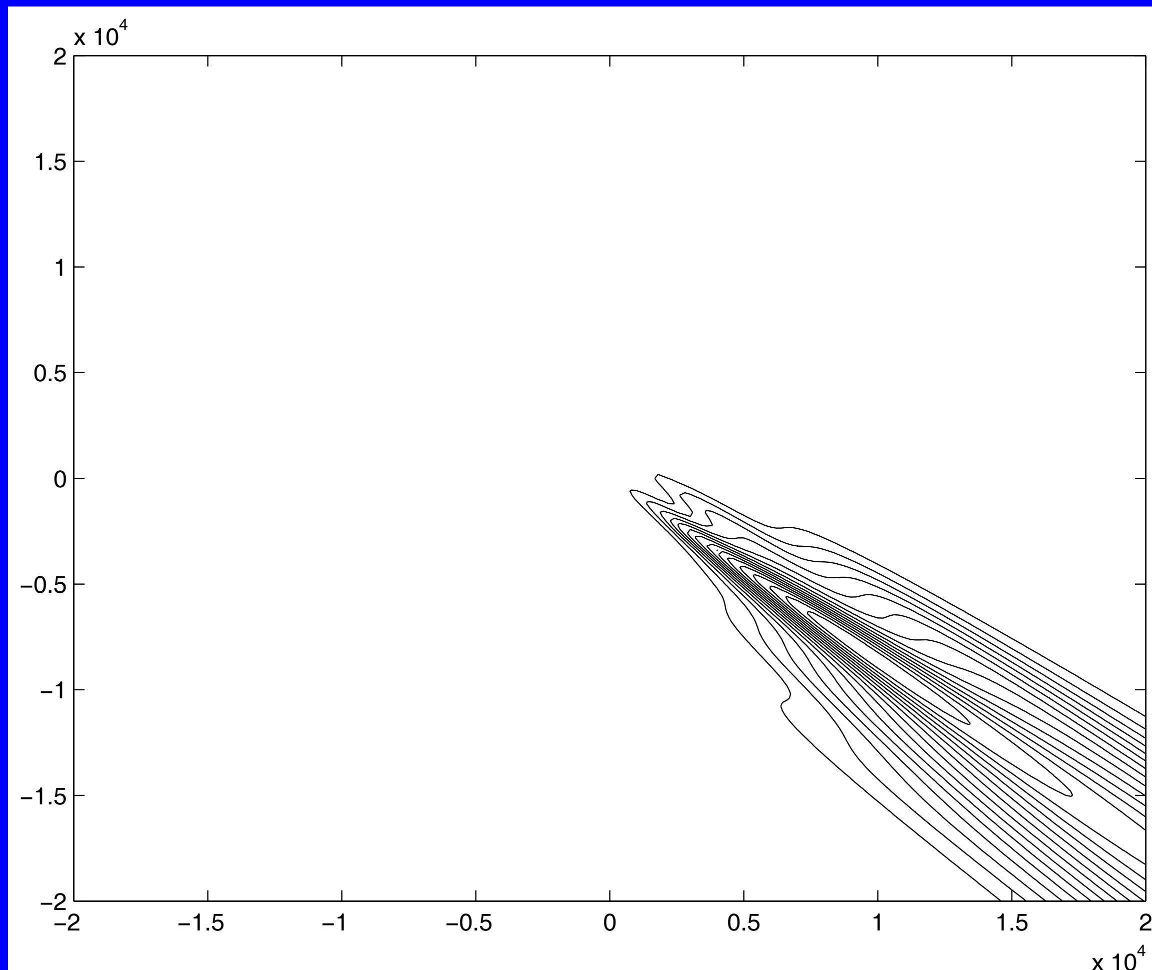
[1]Decree law number 352/90 from 9 November 1990.
[2]Decree law number 111/2002 from 16 April 2002.
[3]Decree law number 286/93 from 12 March 1993.

Numerical results

	Instance 1	Instance 2	Instance 3
h_1	0.00	10.00	196.93
h_2	78.26	69.09	380.06
h_3	0.00	10.00	403.12
h_4	153.17	152.64	428.38
h_5	80.90	71.27	344.81
h_6	0.00	10.00	274.58
h_7	13.52	13.52	402.83
h_8	161.78	161.87	396.82
h_9	141.73	141.63	415.58
h_{10}	15.05	15.05	423.99
Total	644.40	655.06	3667.10

Constraint contour



Example - Maximum pollution level and sampling stations planning (Gustafson *et al.*, 1977)

Computing the maximum pollution level (l^*) by fixing the stack height h_i.

A region with 25 stacks.

The region considered was $\mathcal{R} = [0, 24140] \times [0, 24140]$ (square of about 15 miles).

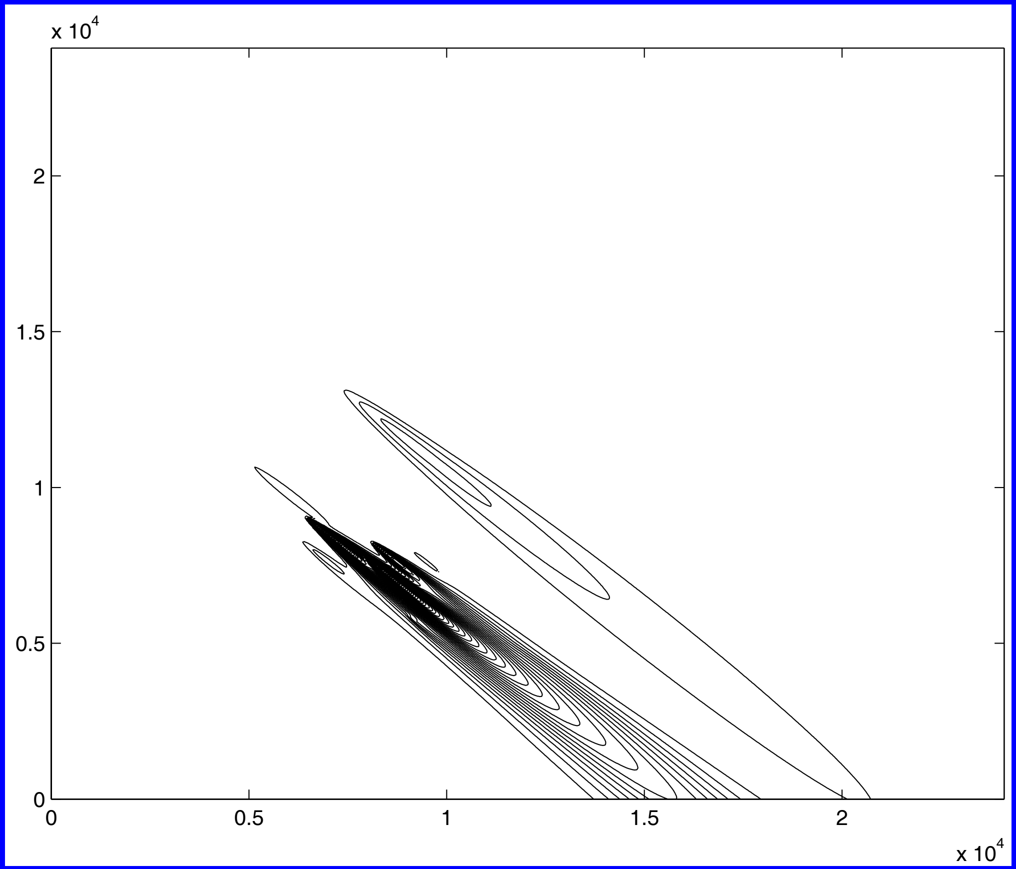
Environment temperature of $284K$, and wind velocity of $5ms^{-1}$ in direction $3.927rad$ (225^o).

Data for the 25 stacks

Source	a_i (m)	b_i (m)	h_i (m)	d_i (m)	$\mathcal{Q}_i$ (gs^{-1})	$(V_O)_i$ (ms^{-1})	$(T_O)_i$ (K)
1	9190	6300	61.0	2.6	191.1	6.1	600
2	9190	6300	63.6	2.9	47.7	4.8	600
3	9190	6300	30.5	0.9	21.1	29.2	811
4	9190	6300	38.1	1.7	14.2	9.2	727
5	9190	6300	38.1	2.1	7.0	7.0	727
6	9190	6300	21.9	2.0	59.2	4.3	616
7	9190	6300	61.0	2.1	87.2	5.2	616
8	8520	7840	36.6	2.7	25.3	11.9	477
9	8520	7840	36.6	2.0	101.0	16.0	477
10	8520	7840	18.0	2.6	41.6	9.0	727
11	8050	7680	35.7	2.4	222.7	5.7	477
12	8050	7680	45.7	1.9	20.1	2.4	727
13	8050	7680	50.3	1.5	20.1	1.6	727
14	8050	7680	35.1	1.6	20.1	1.5	727
15	8050	7680	34.7	1.5	20.0	1.6	727
16	9190	6300	30.0	2.2	24.7	9.0	727
17	5770	10810	76.3	3.0	67.5	10.7	473
18	5620	9820	82.0	4.4	66.7	12.9	603
19	4600	9500	113.0	5.2	63.7	9.3	546
20	8230	8870	31.0	1.6	6.3	5.0	460
21	8750	5880	50.0	2.2	36.2	7.0	460
22	11240	4560	50.0	2.5	28.8	7.0	460
23	6140	8780	31.0	1.6	8.4	5.0	460
24	14330	6200	42.6	4.6	172.4	13.4	616
25	14330	6200	42.6	3.7	171.3	16.1	616

Numerical results - contour

The maximum pollution level of $l^* = 1.81068 \times 10^{-3} gm^{-3}$ in position $(x, y) = (8500, 7000)$.



Example - Air pollution abatement (Gustafson and Kortanek, 1972)

Consider three plants ($\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$), with emissions of e_1, e_2 and e_3, where $0 \le e_i \le 2$, $(i = 1, 2, 3)$ of a certain pollutant. By legal imposition the pollution level must not exceed a given threshold ($\mathcal{C}_0$) under mean weather conditions, i.e., $\theta = 0$ and $\mathcal{U} = \left(\frac{1}{2\pi}\right)^2 ms^{-1}$. Consider $\mathcal{Q} = 1gs^{-1}$ and $\mathcal{C}_0 = \frac{1}{2}$. The remaining stacks data are

Source	a_i	b_i	h_i
1	0	1	1
2	0	0	1
3	2	-1	$\sqrt{2}$

Problem

The emission rate reduction is to be minimized.

$$\min_{r_1,r_2,r_3\in R} 2r_1 + 4r_2 + r_3$$

$$s.t. \sum_{i=1}^{3}(2 - r_i)\mathcal{C}(x, y, 0, \mathcal{H}_i) \leq \mathcal{C}_0$$

$$0 \leq r_i \leq 2,\ i = 1, 2, 3$$

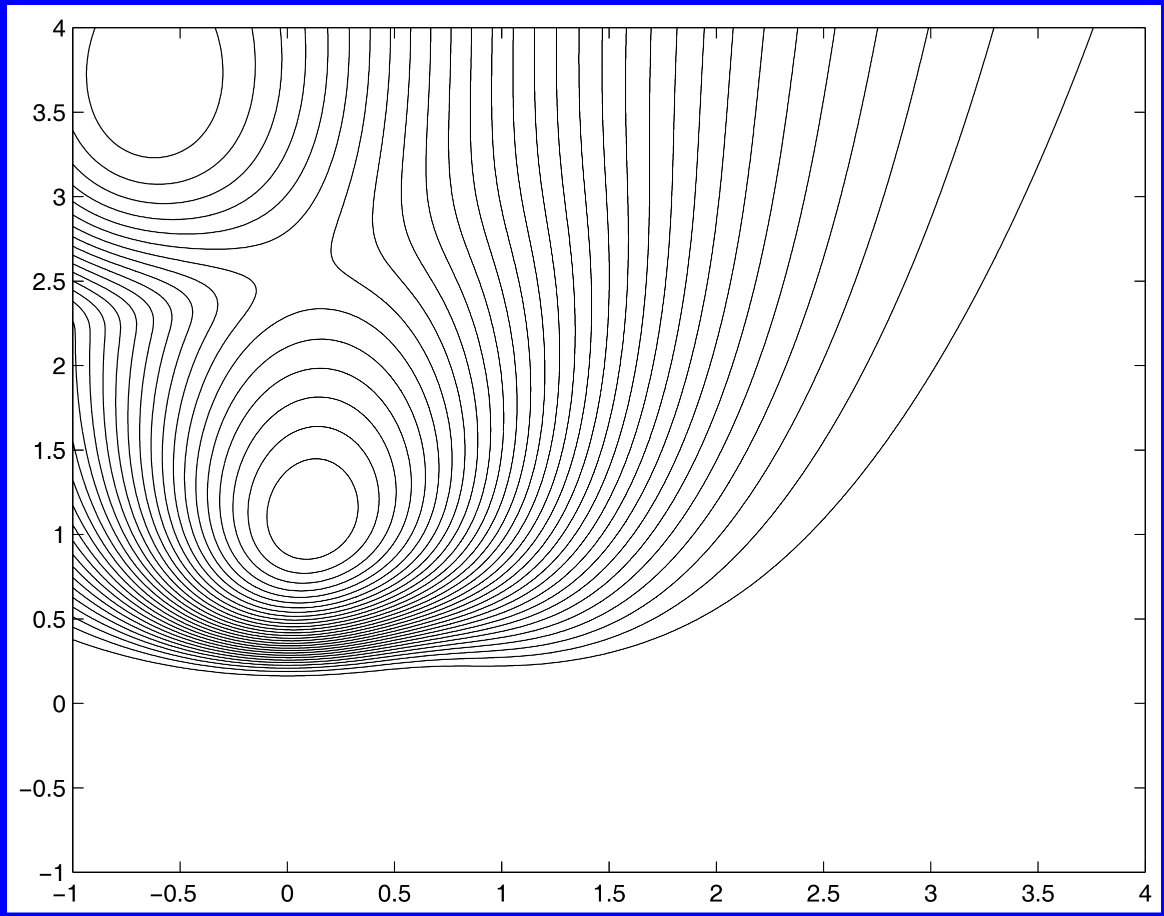
$$\forall(x, y) \in [-1, 4] \times [-1, 4].$$

Numerical results

Solution found $r^* = (0.987, 0.951, 0.943)$

The maximum pollution is attained at $(x, y)^1 = (1.100, 0.125)$, $(x, y)^2 = (1.100, 0.100)$ and $(x, y)^3 = (3.675, -0.625)$, where the sampling stations should be placed.

Constraint contour



Example - Air pollution abatement (Wang and Luus, 1978)

The data proposed by (Gustafson and Kortanek, 1972), in spite of illustrating the air pollution abatement problem, is not a real scenario.

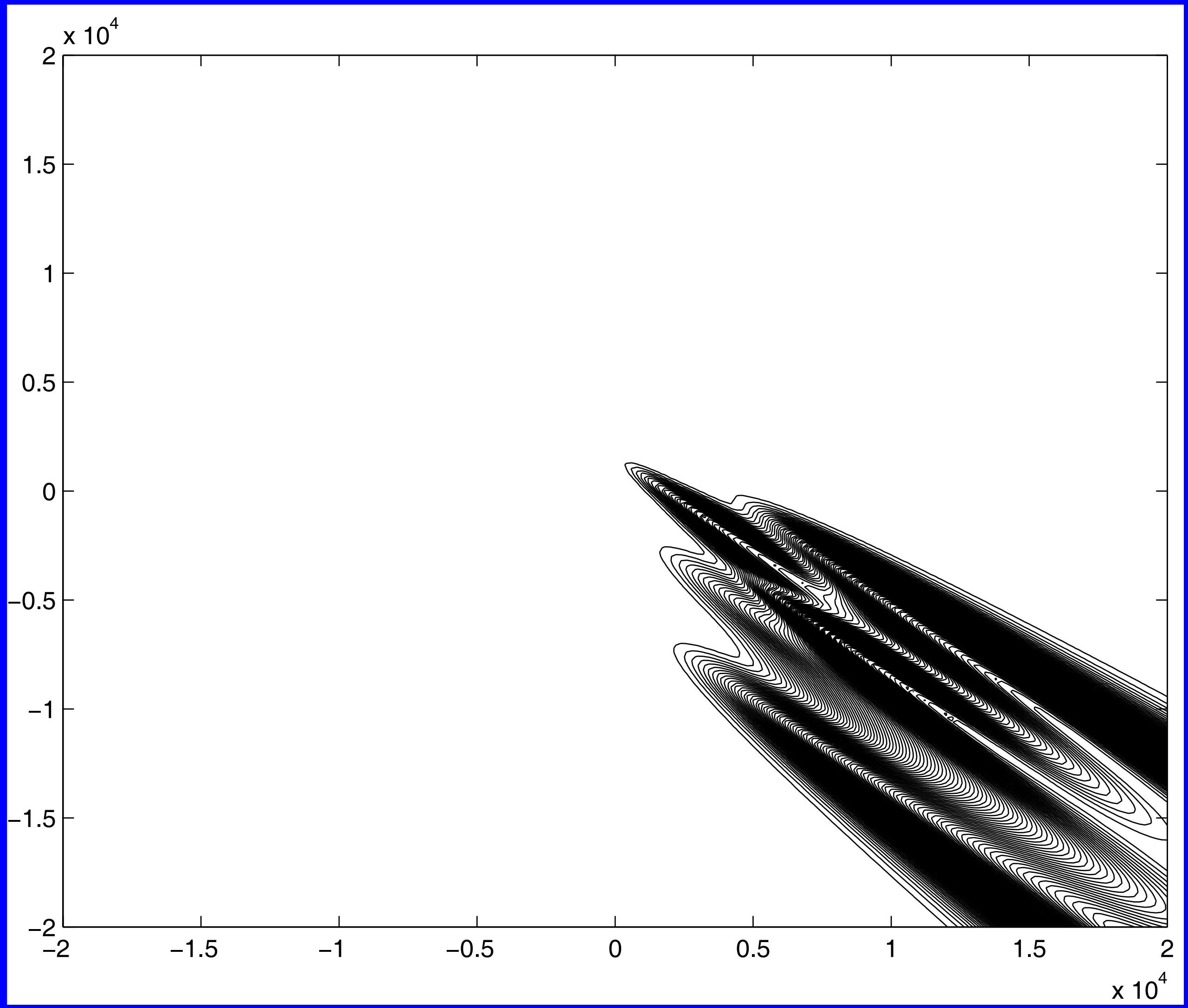
We have used the data from (Wang and Luus, 1978) with the Portuguese limit of $\left(\sum_{i=1}^{10}(1-r_i)\mathcal{C}_i(x,y,0,\mathcal{H}_i) \leq 1.25 \times 10^{-4} gm^{-3}\right)$.

Numerical results

The initial guess is $r_i = 0$, $i = 1, \ldots, 10$, corresponding to no reduction in all sources.

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	Total
0.11	0.61	1	0.69	1	0.23	0.75	0.56	1	1	6.95

Constraint contour



Conclusions

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`vaz1.mod`	Minimum stack height
`vaz2.mod`	Maximum attained pollution and sampling stations planning
`vaz3.mod`	Air pollution abatement
`vaz4.mod`	Air pollution abatement

Publicly available together with the SIPAMPL at

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- Numerical results obtained with the NSIPS solver;

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B.-C. Wang and R. Luus. Reliability of optimization procedures for obtaining global optimum. *AIChE Journal*, 24(4):619–626, 1978.

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email: aivaz@dps.uminho.pt
ecferreira@deb.uminho.pt
Web http://www.norg.uminho.pt/aivaz/
http://www.deb.uminho.pt/ecferreira/

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