A modified differential evolution based solution technique for economic dispatch problems

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Outline of the Presentation

1. Introduction
2. Motivation
3. Economic Dispatch Problem, ED
4. Differential Evolution, DE
5. Our Modified Differential Evolution (mDE)
6. Procedures to Solve ED by mDE
7. Experimental Results
8. Conclusions
Economic dispatch (ED) is important in power generation systems. ED finds optimal dispatch from different units in a given time period. ED minimizes total cost while satisfying specified constraints.

- Incusion of ramp-rate constraints in ED makes the problem dynamic economic dispatch (DED).
- The cost function is quadratic and smooth.
- If valve-point loading effects are considered it becomes nonsmooth.
### Introduction

#### Solution methods: smooth cost function
- Linear programming and non-linear programming algorithm.
- Quadratic programming algorithm.
- Lagrangian relaxation algorithm.

#### Solution methods: nonsmooth cost function
- Genetic algorithm.
- Simulated annealing.
- Particle swarm optimization.
- Evolutionary programming.
- Hybrid stochastic search.
- Augmented Lagrange Hopfield network and
- Differential evolution
Motivation

- Differential Evolution (DE) is proposed by Storn and Price in 1997.
- DE is a population based heuristic approach and has only three parameters.
- DE is be very efficient to derivative free problems.
- Existing solution methods solve ED problem by using penalty function method.
- It is very difficult to find appropriate penalty parameter.
Motivation

Proposed Method

A modified differential evolution (mDE) based solution technique for economic dispatch problems considering

- Valve-point loading effects, transmission loss and ramp-rate constraints.

In mDE,

- Self-adaptive parameters with modified mutation and inversion operator.
- Modified selection based on feasibility.
Economic Dispatch Problem, ED

ED problem

Suppose the power generation systems have $N$ generating units that generate power for $T$ time period.

ED objective function

$$
\text{min } f = \sum_{i=1}^{N} \sum_{t=1}^{T} C_{it}(P_{it}) \tag{1}
$$

where, $P_{it}$ is the power output from unit $i$ at time $t$. $C_{it}(P_{it})$ is the cost of unit $i$ at time $t$.

ED cost function, smooth

$$
C_{it}(P_{it}) = a_i P_{it}^2 + b_i P_{it} + c_i \tag{2}
$$

where, $a_i$, $b_i$ and $c_i$ are the cost coefficients of unit $i$. 
In reality, the objective function of ED has nondifferentiable points due to the valve-point loading effects.

**ED cost function, nonsmooth**

\[ C_{it}(P_{it}) = a_i P_{it}^2 + b_i P_{it} + c_i + |e_i \sin (f_i (P_{i\min} - P_{it}))| \]  \hspace{1cm} (3)

where, \( e_i \) and \( f_i \) are the coefficients of unit \( i \) reflecting valve-point loading effects.
Economic Dispatch Problem, ED

ED minimizes fuel cost associated with \( N \) units in \( T \) time.

Real power balance constraints

\[ \sum_{i=1}^{N} P_{it} = D_t + L_t, \quad t = 1, 2, \ldots, T \]  \hspace{1cm} (4)

where, \( D_t \) is the total assumed load demand at time \( t \).
\( L_t \) is the transmission loss at time \( t \).

Transmission loss

\[ L_t = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{it} B_{ij} P_{jt} \]  \hspace{1cm} (5)

where, \( B \) is the \( N \times N \) loss coefficients matrix.
Generating unit ramp rate limits

\[
P_{it} - P_{i(t-1)} \leq UR_i
\]
\[
P_{i(t-1)} - P_{it} \leq DR_i
\]
\[i = 1, 2, \ldots, N \quad t = 1, 2, \ldots, T
\]

where, \( UR_i \) and \( DR_i \) are the ramp-up and ramp-down limits of \( i \)th unit, respectively.

Real power operating limits

\[
P_{i\text{min}} \leq P_{it} \leq P_{i\text{max}}, \quad i = 1, 2, \ldots, N
\]

where, \( P_{i\text{min}} \) and \( P_{i\text{max}} \) are the minimum and the maximum real power outputs of \( i \)th unit, respectively.
Economic Dispatch Problem, ED

ED problem can be formulated as a constrained nonlinear programming problem

$$\begin{align*}
\min & \quad f(P) \\
\text{s.t.} & \quad h_k(P) = 0 \quad k = 1, 2, \ldots, m_1 \\
& \quad g_k(P) \geq 0 \quad k = 1, \ldots, m_2 \\
& \quad P_{\text{min}} \leq P \leq P_{\text{max}}
\end{align*}$$

(8)

where, \( m_1 = T \) and \( m_2 = 2N \times T \) and hence total number of constraints \( m = m_1 + m_2 \).
Differential Evolution, DE

- DE creates new candidate solutions by combining points of the same population.
- A candidate replaces a current solution only if it has better objective value.
- DE has three parameters:
  1. Amplification factor of the difference vector $F$.
  2. Crossover control parameter $CR$.

DE’s operators

1. Mutation
2. Crossover
3. Selection
Nonlinear optimization problems with simple bounds

\[
\begin{align*}
\min \quad & f(x) \\
\text{subject to} \quad & x \in \Omega 
\end{align*}
\]  

(9)

where, \( f : \mathbb{R}^n \rightarrow \mathbb{R} \) with \( \Omega = \{ x \in \mathbb{R}^n : lb_j \leq x_j \leq ub_j, \quad j = 1, \ldots, n \} \), and \( lb, ub \in \mathbb{R}^n \).

Target Point

- \( n \)-problem dimension, \( NP \)-population size and \( z \)-generation index.
- The target point at \( z = 1 \) is generated randomly

\[
x_{p,1} = lb + r \ast (ub - lb), \quad p = 1, 2, \ldots, NP,
\]

(10)

where, \( r \sim U[0, 1] \).
Outline of Differential Evolution

Mutation

DE creates mutant points in a population by

\[ v_{p,z+1} = x_{r_1,z} + F(x_{r_2,z} - x_{r_3,z}). \]  (11)

- Integer random numbers \( r_1, r_2, r_3 \sim U\{1, 2, \ldots, NP\} \) and \( r_1 \neq r_2 \neq r_3 \neq p \).
- \( F \in [0, 2] \) constant parameter which controls the amplification of the differential variation \( (x_{r_2,z} - x_{r_3,z}) \).

\( NP \) must be greater or equal to 4
Crossover [Trial Point]

The mutant point’s components are then mixed with the target point’s components to yield the so-called trial point $u_{p,z+1}$

$$u_{pj,z+1} = \begin{cases} v_{pj,z+1} & \text{if } (r_j \leq CR) \text{ or } j = z_p \\ x_{pj,z} & \text{if } (r_j > CR) \text{ and } j \neq z_p \end{cases} \quad (12)$$

- Random $r_j \sim U[0, 1]$ performs the mixing of $j$th component of points and $CR \in [0, 1]$ is a constant parameter.
- Random integer index $z_p \sim U\{1, 2, \ldots, n\}$ ensures that $u_{p,z+1}$ gets at least one component from $v_{p,z+1}$. 
Outline of Differential Evolution

Bounds Check

After crossover the bounds of each component must be checked

\[ u_{pj,z+1} = \begin{cases} 
    l_b^j & \text{if } u_{pj,z+1} < l_b^j \\
    u_b^j & \text{if } u_{pj,z+1} > u_b^j \\
    u_{pj,z+1} & \text{otherwise.}
\end{cases} \]  

(13)

Selection

The trial point \( u_{p,z+1} \) is compared to the target point \( x_{p,z} \) to decide whether or not it should become a member of generation \( z + 1 \) as

\[ x_{p,z+1} = \begin{cases} 
    u_{p,z+1} & \text{if } f(u_{p,z+1}) \leq f(x_{p,z}) \\
    x_{p,z} & \text{otherwise.}
\end{cases} \]  

(14)
Our Modified Differential Evolution (mDE)

- Our mDE includes modifications proposed by Brest et al. (2006) and Kaelo et al. (2006).

- Control parameters \((F_p, CR_p)\) are calculated by

\[
F_{p,z+1} = \begin{cases} 
F_l + \lambda_1 \times F_u, & \text{if } \lambda_2 < \tau_1 \\
F_{p,z}, & \text{otherwise}
\end{cases}
\]  

\[
CR_{p,z+1} = \begin{cases} 
\lambda_3, & \text{if } \lambda_4 < \tau_2 \\
CR_{p,z}, & \text{otherwise.}
\end{cases}
\]

- Random \(\lambda \sim U[0, 1]\) and \(\tau_1 = \tau_2 = 0.1\).

- \(F_l = 0.1\) and \(F_u = 1.0\). So \(F_{p,z+1} \in [0.1, 1.0]\) and \(CR_{p,z+1} \in [0, 1]\).
Our Modified Differential Evolution (mDE)

- Kaelo et al. proposed alternative technique for mutation

\[
v_{p,z+1} = x_{1,z} + F(x_{2,z} - x_{3,z})
\]

\[
x_{1,z} = \arg \min \{ f(x_{r1,z}), f(x_{r2,z}), f(x_{r3,z}) \}.
\]

- After every \( B \) generations we use best point found so far as the base point

\[
v_{p,z+1} = x_{best,z} + F(x_{r1,z} - x_{r2,z}).
\]
Our Modified Differential Evolution (mDE)

Inversion operator

Our mDE has inversion operator with some inversion probability ($p_{\text{inv}} \in [0, 1]$) to points after crossover.

Illustrative example of inversion:

\[
u_{p,z} = \begin{bmatrix}
    u_{p1,z} & u_{p2,z} & u_{p3,z} & u_{p4,z} & u_{p5,z} & u_{p6,z} & u_{p7,z} & u_{p8,z}
\end{bmatrix}
\]

\[
u'_{p,z} = \begin{bmatrix}
    u_{p1,z} & u_{p2,z} & u_{p6,z} & u_{p5,z} & u_{p4,z} & u_{p3,z} & u_{p7,z} & u_{p8,z}
\end{bmatrix}
\]
## Our Modified Differential Evolution (mDE)

### Termination condition

- $z$: current generation
- $G_{\text{max}}$: maximum generations.

The termination condition is:

$$((z > G_{\text{max}}) \text{ OR } (|f_{\text{max}, z} - f_{\text{min}, z}| \leq \epsilon (= 10^{-6}))))$$

- $f_{\text{max}, z}$: max. function value
- $f_{\text{min}, z}$: min. function value
Procedures to Solve ED by mDE

Array of power output variables

Let $P_{it}$ be the power output from unit $i$ at time $t$

$$P = \begin{bmatrix}
P_{11} & P_{12} & \ldots & \ldots & \ldots & P_{1T} \\
P_{21} & P_{22} & \ldots & \ldots & \ldots & P_{2T} \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
P_{N1} & P_{N2} & \ldots & \ldots & \ldots & P_{NT}
\end{bmatrix}. \tag{17}$$

Initialization of Individuals

An individual $P$ has $(N \times T)$ components at $z$ are initialized

$$P_{it} = P_{i \text{min}} + \delta \ast (P_{i \text{max}} - P_{i \text{min}}), \tag{18}$$

where, $\delta \sim U[0, 1], i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$. 
Equality Constraints: Real Power Balance

To satisfy (4), select a dependent power output $P_{lt}$, and is computed from (19)

$$P_{lt} = D_t + L_t - \sum_{i=1,i\neq l}^{N} P_{it}, \quad t = 1, 2, \ldots, T. \quad (19)$$

The transmission loss $L_t$ is a function of all units including that of dependent unit, and is given by

$$L_t = \sum_{i=1,i\neq l}^{N} \sum_{j=1,j\neq l}^{N} P_{it} B_{ij} P_{jt} + 2P_{lt} \left( \sum_{i=1,i\neq l}^{N} B_{li} P_{it} \right) + B_{ll} P_{lt}^2. \quad (20)$$
Substituting $L_t$ from (20) to (19), and equation (19) becomes

\[
B_{ll} P_{lt}^2 + \left( 2 \sum_{i=1,i \neq l}^{N} B_{li} P_{it} - 1 \right) P_{lt} \\
+ \left( D_t + \sum_{i=1,i \neq l}^{N} \sum_{j=1,j \neq l}^{N} P_{it} B_{ij} P_{jt} - \sum_{i=1,i \neq l}^{N} P_{it} \right) = 0.
\]  
(21)

The above equation (21) can be written by

\[
AP_{lt}^2 + BP_{lt} + C = 0.
\]
(22)

Then the value of $P_{lt}$ can be obtained by solving (22).
Procedures to Solve ED by mDE

Inequality Constraints

Constraints violation of an individual point are calculated by \( \min \{0, g_k(P)\}, k = 1, 2, \ldots, m_2 \). The sum of constraints violation is calculated by

\[
\Gamma = \sum_{k=1}^{m_2} |g_k(P)|. \tag{23}
\]

Fitness calculation

\[
\Phi(P) = \begin{cases} 
 f(P) & \text{if } g_k(P) \geq 0 \quad \forall k = 1, 2, \ldots, m_2 \\
 f_{\text{max } \text{feasi}} + \Gamma & \text{otherwise}
\end{cases} \tag{24}
\]

where, \( f_{\text{max } \text{feasi}} \) is the objective function value of the worst feasible solution in the population.
Procedures to Solve ED by mDE

Modified Selection in mDE

In mDE we propose the selection of an individual point at generation $z + 1$ based on feasibility of that individual point Deb(2000). Let $P_z$ and $P_{z+1}$ be a target point and trial point, respectively. To select which point will be the member of population at $z + 1$, we use the following criteria:

1. Any feasible point is preferred to any infeasible point.
2. Between two feasible points, one having better objective function value is preferred.
3. Between two infeasible points, one having smaller constraints violation is preferred.
Experimental Results

We coded mDE in C and compiled with Microsoft Visual Studio 9.0 compiler in a PC having 2.5 GHz Intel Core 2 Duo processor and 3 Gb RAM. We set $\epsilon = 10^{-6}$, $NP = \min(100, 10n)$, $B = 10$, $F_I = 0.1$, $F_U = 1.0$, $\tau_1 = \tau_2 = 0.1$ and $p_{\text{inv}} = 0.05$.

P4

This ED problem has 40 generating units with hourly demand of 10500 and smooth cost function. We set $G_{\text{max}} = 3000$. The results obtained by mDE and from literature are shown in Table 1.

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<th>SA</th>
<th>GA</th>
<th>HDE</th>
<th>VSHDE</th>
<th>ALHN</th>
<th>mDE</th>
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<td>144486.02</td>
<td>143955.83</td>
<td>143943.90</td>
<td>143926.90</td>
<td>141127.40</td>
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Experimental Results

Figure: Profile of $f_{\text{min}}$ at different generations
Experimental Results

This DED problem has 5 generating units with 24 hours demands and nonsmooth cost function with transmission loss. We set $G_{max} = 10000$. The results obtained by mDE and from literature are shown in Table 2.

**Table:** Comparison of results of P5

<table>
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<th>DE</th>
<th>mDE</th>
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</thead>
<tbody>
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<td>min. cost</td>
<td>47356.00</td>
<td>45800.00</td>
<td>43213.00</td>
<td>43057.83</td>
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Experimental Results

Figure: Profile of $f_{\text{min}}$ at different generations
Conclusions

- A modified differential evolution based solution technique for economic dispatch problems is presented.
- The ED problem with valve-point loading effect and transmission loss is considered.
- The mDE with self-adaptive parameters and modified mutation with inversion is proposed.
- In mDE, modified selection technique based on feasibility is proposed.
- A comparative study based on obtained results found by mDE and in literature is presented.
- It is shown that our mDE outperformed other methods found in literature.
References


Thank You Very Much